Evaluation of the hydraulic conductivity of aquitards

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The evaluation of the bulk vertical hydraulic conductivity of an aquitard based on its response to the pumping of an adjacent aquifer is examined using Biot's theory. Consideration is given to the errors in interpretation of the results of pumping tests which arise as a result of the time lag associated with different types of piezometers as well as the length of the piezometer. Factors to allow for correction of these errors are presented. Although these factors are originally developed for isotropic aquitards, they can be used for anisotropic aquitards with appropriate modifications described in the paper. A comparison is made between the results obtained from diffusion theory (as assumed in the development of techniques currently used in practice) and the more rigorous Biot's theory. The application of the technique is illustrated by two examples.

Key words: hydraulic conductivity, field test, analysis, pumping test, piezometers, anisotropy.

L'évaluation de la conductivité hydraulique verticale de masse d'un aquitard basée sur sa réaction au pompage d'un aquifère adjacent est examinée à la lumière de la théorie de Biot. L'on prend en considération les erreurs dans l'interprétation des résultats des essais de pompage qui découlent du temps de réponse des différents types de piézomètres de même qu'à leur longueur. Les facteurs pour permettre de corriger ces erreurs sont présentés. Quoique ces facteurs aient été développés à l'origine pour des aquitards isotropes, ils peuvent être utilisés pour des aquitards anisotropes avec les modifications appropriées décrites dans l'article. L'on compare les résultats obtenus par la théorie de diffusion (telle que supposée dans la mise au point de techniques couramment utilisées en pratique) et par la théorie plus rigoureuse de Biot. L'application de la technique est illustrée par deux exemples.

Mots clés : conductivité hydraulique, essai sur le terrain, analyse, essai de pompage, piézomètres, anisotropie.

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Introduction

To assess the potential impact of a waste-disposal facility which is to be built in an aquitard overlying an aquifer, it is first necessary to evaluate the bulk vertical hydraulic conductivity of the aquitard. Conventional hydraulic conductivity tests (e.g., rising-, falling-, and constant-head tests; Hvorslev 1951; and many others) primarily provide information concerning horizontal hydraulic conductivity. As a consequence, it is becoming increasingly common to assess the bulk vertical hydraulic conductivity of the aquitard using results obtained from a pumping test on an adjacent aquifer (e.g., Grisak and Cherry 1975; Rodrigues 1983; Keller et al. 1986).

The common method of estimating the bulk hydraulic conductivity of an aquitard involves (*i*) installing a pumping well into the aquifer, (*ii*) installing adjacent observation wells in the aquifer and the aquitard at some distance r from the pumping well (see the insert to Fig. 1), (*iii*) monitoring the drawdown s with time in the aquifer piezometer (relative to the initial static water level in that piezometer), (*iv*) monitoring the drawdown s' with time in the aquitard piezometer (relative to the initial static water level in that piezometer), and (*v*) using the ratio (s'/s) of the drawdown in the aquitard piezometer to the drawdown in the aquifer piezometer at a given time to estimate the bulk hydraulic conductivity of the aquitard. The technique for doing this is known as the ratio method (Neuman and Witherspoon 1972). The objective of the present paper is to discuss some of the practical limitations of the ratio method for interpreting the hydraulic conductivity and to present a modification to the approach which addresses these limitations.

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The ratio method

The principle behind the ratio method can be described as follows. The piezometer located in the aquifer responds relatively quickly to pumping from the pumping well. This change in head in the aquifer causes a change in pore pressure in the aquitard adjacent to the aquifer. As a consequence, a transient "pressure wave" passes through the aquitard as the pore pressures in the aquitard adjust to the changed conditions in the aquifer. The rate at which this pressure wave migrates through the aquitard is a function of the consolidation characteristics of the aquitard, and these are a function of the compressibility and bulk hydraulic conductivity of the aquitard. Thus, there will often be a significant period of time (100s to 1000s of minutes) between the time when there is a change in head in the aquifer and the subsequent initial measurable response in the aquitard. Since this pressure wave is migrating vertically through the aquitard under approximately one-dimensional conditions, the time at which a monitor in the aquitard responds to the pumping of the aquifer is primarily controlled by the vertical hydraulic conductivity and compressibility of the aquitard. Based on this observation and diffusion theory, Neuman and Witherspoon (1972) developed theoretical solutions for an ideal piezometer which responds without any time lag due to the piezometer characteristics and which has zero length. These solutions related the drawdown ratio s'/s of the aquitard–aquifer piezometers to a dimensionless time factor t'_D (see Fig. 1), where

$$[1a] \quad t'_D = \frac{K' t^*}{S'_s (z^*)^2}$$

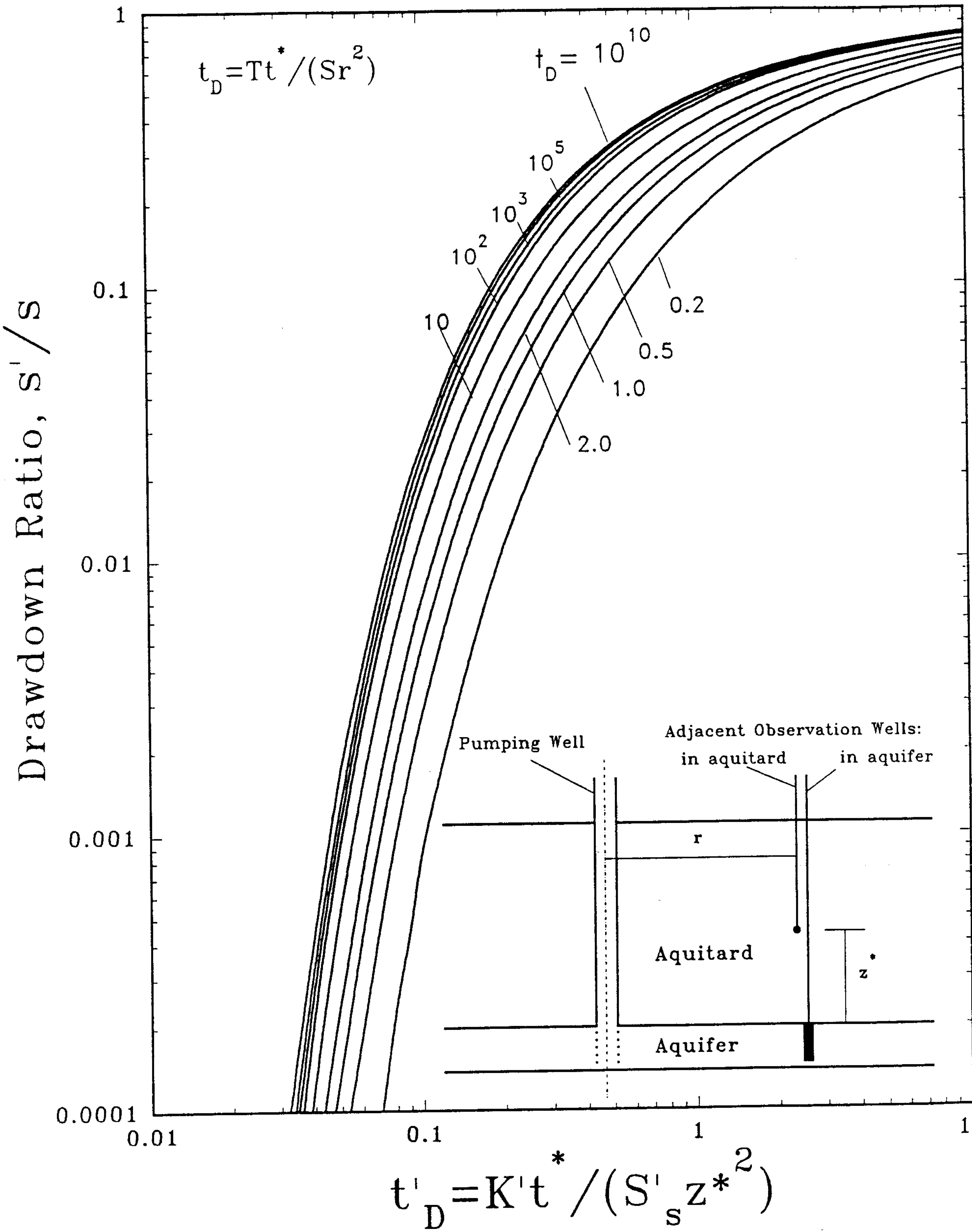

FIG. 1. Variation of drawdown ratio s'/s with dimensionless time factor t'_D for semi-infinite aquitard (modified from Neuman and Witherspoon 1972). K', aquitard hydraulic conductivity; T, aquifer transmissivity; s', aquitard drawdown; s, aquifer drawdown; t^*, elapsed time; S, storage coefficient of aquifer; s_s, specific storage of aquitard; t_D, dimensionless time factor for the aquifer; t_D, dimensionless time factor for the aquitard.

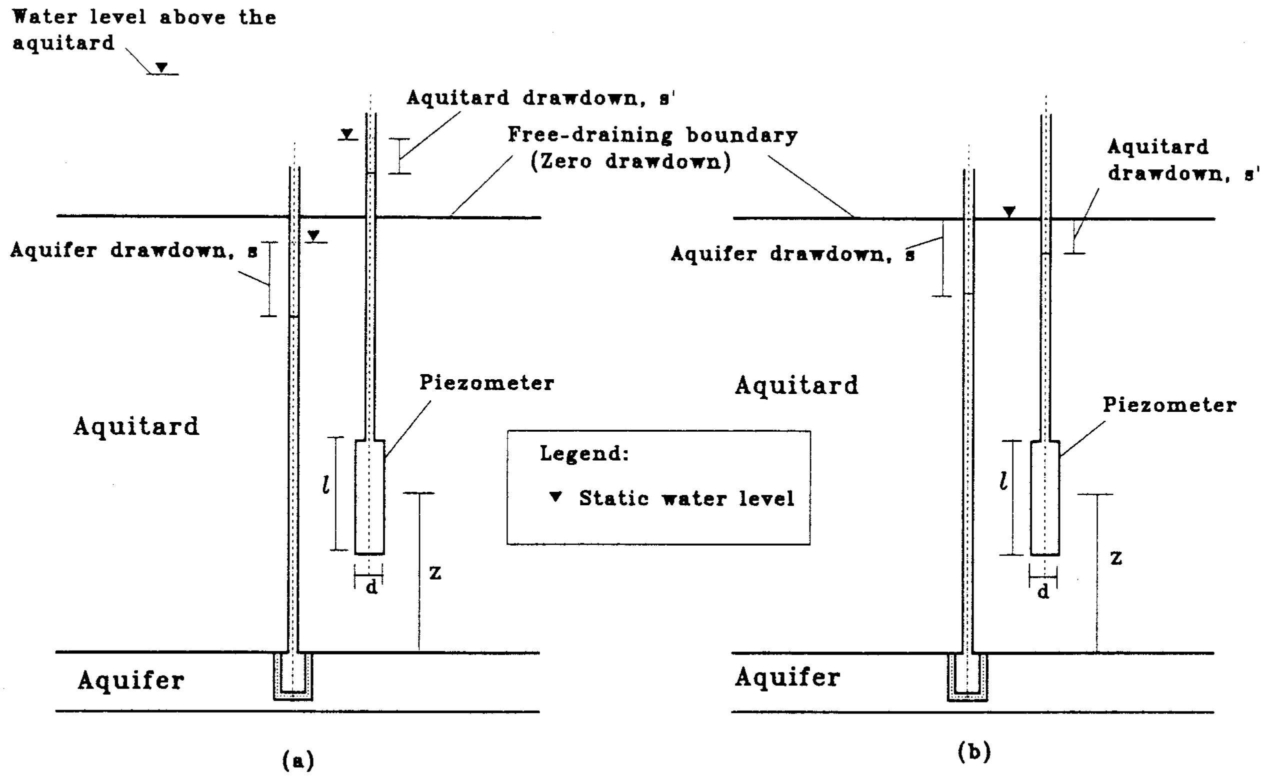

FIG. 2. Schematic view of the piezometer and aquifer–aquitard system. Solution of the consolidation analysis is independent of static water levels. (*a*) Downward flow under static conditions. (*b*) Hydrostatic initial condition.

and where t'_D is dimensionless time
K' is vertical bulk hydraulic conductivity of the aquitard
t^* is the time corresponding to the drawdown ratio s'/s for an ideal piezometer
z^* is the distance from the aquifer to the ideal (point) piezometer in the aquitard
S'_s is the specific storage of the aquitard $= m_v\gamma_w$
m_v is a coefficient of volume change
γ_w is the unit weight of water.

The term K'/S'_s is the hydraulic diffusivity of the aquitard and is equivalent to the coefficient of consolidation under one-dimensional (1D) strain conditions, $C_v = C_1 = K'/S'_s$, if the compressibility of water is negligible (Kashef 1986).

The relationship between t'_D and s'/s also depends on the aquifer properties as expressed in terms of a second dimensionless factor t_D where

$$[1b] \quad t_D = \frac{Tt}{(Sr^2)}$$

and where T is transmissivity of the aquifer
S is storage coefficient of the aquifer
t is elapsed time ($t = t^*$ for an ideal piezometer)
r is radial distance of piezometer from the pumping well.

Based on the aquifer response, the location of piezometer (r), and the time of interest (t), the dimensionless time t_D can be calculated, and hence for a given drawdown ratio (s'/s) the dimensionless time t'_D can be obtained from Fig. 1.

Having established t'_D, the hydraulic conductivity K' can be deduced if the distance z^* from the aquifer to the (ideal) monitor and the specific storage S'_s are known. The value of hydraulic conductivity K' deduced is based on the assumed use of an ideal point source piezometer which will respond immediately to a change in pressure head in the adjacent soil, and the assumption that diffusion theory is adequate.

Limitations associated with direct use of Neuman and Witherspoon's ratio method

Ideally, vibrating-wire pressure transducers could be used to assess the pressure changes in the aquitard with a minimum time lag. However, in practice, the drawdown observations are sometimes made using simple piezometers (e.g., standpipes or observation wells) which have a relatively high hydrodynamic time lag. Thus the first objective of this paper is to examine the potential effect of the time lag related to the use of piezometers upon the interpreted bulk hydraulic conductivity of the aquitard when using the ratio method.

A second idealization associated with the ratio method (Neuman and Witherspoon 1972) is that of a point piezometer. In reality, piezometers are of finite length and since the transient pressure wave is not linear with depth (especially at early times), the drawdown of a piezometer of finite length may not correspond to the drawdown value at its midlength. Since in the ratio method the hydraulic conductivity is a function of z^2 (where z is the distance of the representative point above the aquifer at which the drawdown is monitored), it is important to assess the potential error associated with the finite length and consequent choice of z. It may be anticipated that the error thus created may be significant especially when interpreting the readings of a relatively long piezometer or of a piezometer installed close to the aquifer. For example, Rodrigues (1983) used an observation well with a well screen about 50 m long at the middepth in an aquitard 100 m thick. He questioned whether the observed drawdowns were representative of the midlength

(z = 50 m) or of the bottom of the observation well (z = 25 m). In his case, the inferred hydraulic conductivity for z = 25 m would be four times smaller than that for z = 50 m. Thus a second objective of this paper is to examine the effect of the finite length of the piezometer and its location relative to the pumped aquifer.

Neuman and Witherspoon's (1972) basic curves for the ratio method (Fig. 1) were developed assuming an isotropic aquitard. They also examined the effect of anisotropy for values of ratios of horizontal to vertical hydraulic conductivity between 1 and 250 and found that because the consolidation was controlled by the vertical hydraulic conductivity, this level of anisotropy had no significant effect on the results. However, this conclusion is not valid for a piezometer of finite length in an anisotropic soil because of the radial flow into the piezometer. The third objective of this paper is to propose a method of estimating the effect of anisotropy on the interpretation of the bulk vertical hydraulic conductivity.

Finally, Neuman and Witherspoon's (1972) ratio method was based on analyses performed using diffusion theory. Although this approach is commonly adopted in estimating transient pore-pressure changes in soil, it neglects the effect of the corresponding effective stress changes on the soil response. This limitation can be overcome using the more complete Biot's theory (Biot 1941; Davis and Poulos 1972; Booker and Small 1975). Thus the final objective of this study is to examine the effect of consolidation of the soil skeleton when assessing the pore-pressure change due to aquifer pumping.

Details of the analyses and assumptions

The basic configuration considered in the analysis is shown in Fig. 2. For reasons of clarity in the following discussion, it is assumed that the aquifer to be pumped is located below the aquitard and is overlain by some more permeable unit (which may be another aquifer or simply a more fractured and weathered portion of the aquitard). The pumping of an aquifer will lower the pressure head adjacent to the aquitard, and this will cause consolidation of the aquitard. The changes in pore pressure that occur in the aquitard can be modelled using consolidation theory.

The boundary conditions for this consolidation analysis relate to changes in pore pressure. Thus at the lower boundary with the pumped aquifer the change in pore pressure is specified to correspond to the drawdown s in the aquifer. At the upper boundary, the change in pore pressure is specified to be zero. The consolidation analysis can then be used to predict the consequent changes in pore pressure with time in the aquitard and hence the drawdown s' in the aquitard piezometer at different times. Since the analysis only depends on the aquitard's physical characteristics (i.e., S_s', K'), piezometer geometry, and changes in pore pressure at the boundaries, it follows that the initial flow conditions are of no significance. Thus, it does not matter whether there was initially downward flow from the upper aquifer to the lower aquifer, as implied by Fig. 2a, or hydrostatic conditions (Fig. 2b) or upward flow from the lower "aquifer" (not shown), provided that the initial static water levels are known.

Although the discussion is focussed on the pumping of a confined aquifer below the aquitard of interest, the results obtained in this paper would be equally valid if the aquifer being pumped is above the aquitard being monitored (provided that z is now measured down from the top of the aquitard). This is because the consolidation of the aquitard will occur in response to a change in head in any adjacent aquifer (irrespective of whether it is above or below the aquitard). However, it is often both easier and of greater practical interest to perform the pumping test on the confined aquifer which is beneath the aquitard of interest, since this is the aquifer that is to be protected from contamination by the aquitard being tested. The results of the present analysis will be valid provided that the changed head in the aquifer approaches a relatively constant value in a short time compared with the time required for the consequent pore-pressure redistribution to move to the piezometer in the aquitard.

Neuman and Witherspoon (1972) examined the effect of the rate of drawdown of the aquifer with time and this effect is considered in the use of the ratio method in terms of t_D (see eq. [1b] and Fig. 1). For many practical problems t_D is large and the drawdown of the aquifer can be approximated by a step function. Since the primary purpose of the present paper is to examine the effect of piezometer time lag and piezometer length, the analyses performed herein assumed that the drawdown of the aquifer could be modelled by a step function. This is similar to the approach adopted by Wolff (1970) and is valid for large values of t_D. Thus, strictly speaking, the results presented in this paper are correct for the case where t_D is large (i.e., as in most practical cases); however, they can also be approximately used as a correction to Neuman and Witherspoon (1972) for all values of t_D given in Fig. 1.

The construction of the pumping well (i.e., whether it is fully screened, fully penetrating, etc.) is important with respect to the determination of aquifer properties (e.g., transmissivity T and storage coefficient S) used to establish the dimensionless aquifer time parameter t_D (see eq. [1b] and Fig. 1). These parameters may be estimated using conventional techniques (e.g., see Freeze and Cherry 1979). In many practical situations, t_D is large and hence precise determination of S and T is not essential, since reasonable uncertainty regarding these values has little effect on the aquitard response (see Fig. 1). However, it is important that the piezometer monitoring the drawdown s of the aquifer near the aquitard piezometer (which monitors s') be screened close to the aquitard so that the response of this aquifer peizometer reflect the change in head adjacent to the aquitard.

The drawdown at the top of the aquitard was assumed to be zero for all time. This assumption is valid (for all layer thicknesses) provided the time of interest is sufficiently small such that the time-dependent pressure response in the aquitard (due to the pumping of the underlying aquifer) has not reached the top of the aquitard. Because of the economic constraint imposed by the length of the pump test, this assumption is commonly satisfied in practice, since one is normally concerned with the early time response of the piezometer in the aquitard.

The heads monitored in piezometers may vary with changes in atmospheric pressure. Thus the changes in atmospheric pressure should be monitored and the piezometer reading corrected for these changes before using the head drops s and s' in the ratio method.

In response to the drawdown in the aquifer, pore water in the aquitard will seep vertically downwards to the aquifer, causing the soils in the aquitard to consolidate. If a piezometer has been installed in the aquitard, then water would, in turn, be drawn from the piezometer into the surrounding soil. The resulting compression of the soil skeleton and changes in the pore pressure are analyzed using Biot's consolidation theory (Biot 1941; Davis and Poulos 1972; Booker and Small 1975). In Biot's theory the soil skeleton is treated as a porous elastic solid and the pore water is coupled to the solid by conditions of compressibility of the soil skeleton and continuity of flow. The pore water is assumed to be incompressible. The three-dimensional (3D) flow problem is simplified into a two-dimensional (2D) axisymmetric problem, and the governing equations are solved using the finite-element method.

Basic finite-element equations

The key equations involved in the finite-element formulation of Biot's theory (based on Small et al. 1976) are given below. The equilibrium equation together with the constitutive relationship can be written as

[2*a*] $K_E \boldsymbol{\delta} - \boldsymbol{L}^T \boldsymbol{U} = \boldsymbol{m}$

and the continuity equation can be written as

[2*b*] $-L\dfrac{d\boldsymbol{\delta}}{dt} - \phi \boldsymbol{U} = 0$

where displacement vector $\boldsymbol{\delta}$ of the soil skeleton and the excess pore pressure vector $\boldsymbol{U}$ are the basic variables to be determined, K_E is the elastic stiffness matrix, ϕ is the fluid stiffness matrix, L is the coupling matrix, and $\boldsymbol{m}$ is the load vector.

Equations [2*a*] and [2*b*] can be integrated if the initial values of $\boldsymbol{U}$ and $\boldsymbol{\delta}$ are known. Suppose that the solution $(\boldsymbol{\delta}_1, \boldsymbol{U}_1)$ is known at time t_1 and it is required to evaluate the solution $(\boldsymbol{\delta}_2, \boldsymbol{U}_2)$ at time $t_2 = t_1 + \Delta t$. Equation [2*b*] can be integrated using the conventional finite difference approximation (Small et al. 1976) to give

[3] $L(\boldsymbol{\delta}_2 - \boldsymbol{\delta}_1) - \phi(\alpha \boldsymbol{U}_2 + (1 - \alpha)\boldsymbol{U}_1)\,\Delta t = 0$

where α defines the nature of the finite difference integration scheme used (e.g., forward difference, central difference; see Small et al. 1976).

Equations [2*a*] and [3] can now be written in the form

[4] $$\begin{bmatrix} K_E & -L^T \\ -L & -\alpha\Delta t\phi \end{bmatrix}\begin{bmatrix} \delta_2 \\ U_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{m}_2 \\ -L\delta_1 + (1-\alpha)\Delta t\phi U_1 \end{bmatrix}$$

Thus, if the values of $\boldsymbol{U}$ and $\boldsymbol{\delta}$ are known at t_1, they can be found at t_2, and so the solution can be marched forward in time. The stability of this integration scheme has been examined by Booker and Small (1975), who demonstrated that the process is unconditionally stable provided $\alpha \geq ½$.

The governing equation (eq. [4]) must be solved subject to appropriate initial and boundary conditions. Prior to initiation of the pump test it was assumed that the pore pressures were in hydrostatic equilibrium (i.e., there were no initial excess pore pressures at time $t = 0$), viz.

[5] $U_0 = 0$

The bottom of the aquitard was subjected to a step change in head s in the underlying aquifer.

[6*a*] $U = -\gamma_w s \quad \text{at} \quad z = 0$

where γ_w is unit weight of water. As discussed in the previous section, this directly corresponds to the Neuman and Witherspoon (1972) case for large values of dimensionless time t_D. The top of the aquitard was assumed to be a free-draining aquifer (i.e., having a hydraulic conductivity several orders of magnitude larger than that of the aquitard), and hence it was assumed that there would not be any change in pore pressure at this boundary due to pumping of the lower aquifer. Thus the excess pore pressure at the top of the aquitard is zero at all times, viz.

[6*b*] $U = 0 \quad \text{at} \quad z = H$

where H is the actual thickness of the aquitard. For times at which the pressure response in the aquitard has not arrived at any point close to the top ($z = H$) boundary, this boundary condition will not have any effect on the results, and hence the actual thickness H of the aquitard has no effect on the interpretation of the results.

The lateral boundary away from the piezometer was assumed to be impermeable. The piezometer was centred on a line of axisymmetry which is a no flow boundary; however, a special boundary condition was developed and implemented to model the piezometer itself as described below.

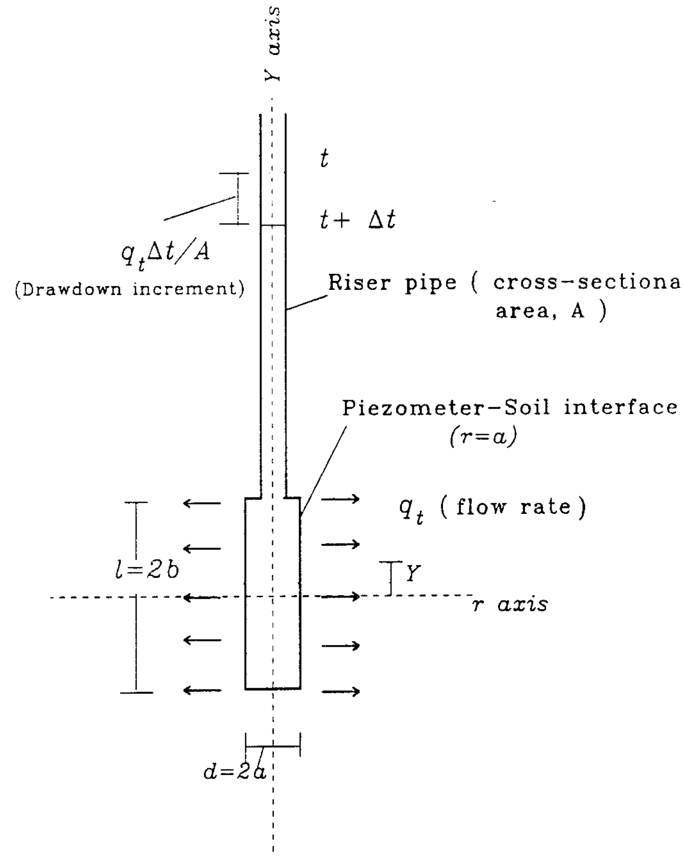

FIG. 3. Sketch showing the relationship between the flow, at the piezometer–soil interface, and the drawdown.

Continuity of flow at the piezometer–soil interface

In the finite-element modelling, a boundary condition must be established at the piezometer–soil interface. Assuming that the piezometer (and sand–gravel pack, if there is one) is saturated, the flow rate q_t across the interface should be equal to the water volume change in the piezometer riser pipe (see Fig. 3). Hence the continuity equation for an isotropic aquitard is

$$[7] \quad 2\pi \frac{K'}{\gamma_w} \int_{-b}^{b} \frac{\partial U}{\partial r} \mathrm{d}Y \Big|_{r=a} = V \frac{\partial U}{\partial t} \Big|_{r=a}$$

where U is the excess pore pressure
r is radial distance from the piezometer axis
K' is hydraulic conductivity of the soil
γ_w is unit weight of water
a is radius of the piezometer
t is elapsed time
$b = l_2$ is half the length of the piezometer
V is a volume factor of the piezometer, and
Y is vertical distance from the centre of the piezometer.

The left hand expression of [7] is the flow rate q_t crossing the piezometer–soil interface, and the right-hand side corresponds to the volume change in the riser pipe.

The volume factor V is the volume of water entering the piezometer for a unit pressure change around the piezometer tip. For a conventional piezometer the volume factor V can be expressed in terms of the cross-sectional area A of the riser pipe, as $V = A/\gamma_w$. For a standpipe where the cross-sectional area is uniform over the entire length, this reduces to $V = \pi a^2/\gamma_w$.

Equation [7] can be rewritten as

$$[8a] \quad q_t = V\left(\frac{U_{t+\Delta t} - U_t}{\Delta t} \right)\Bigg|_{r=a}$$

where

$$[8b] \quad q_t = 2\pi a \frac{K'}{\gamma_w} \int_{-b}^{b} \frac{\partial U}{\partial r} \mathrm{d}Y \Big|_{r=a}$$

Equation [8a] can be rearranged (at $r = a$) to give

$$[9] \quad U_{t+\Delta t} = \frac{q_t}{V} \Delta t + U_t$$

The excess pore pressure at the interface would change with time according to [9], and this boundary condition is incorporated into [4]. The flow rate q_t in [9] can be obtained by numerical integration based on [8b].

Numerical details

The computer program GIBPLT was used in the analyses. This program was developed by significantly modifying the finite element program CONS written by J.C. Small in the 1970s to analyze the consolidation behaviour of soils under axisymmetrical loading. The coding and numerical implementation were checked against benchmark cases including the analytical solutions, based on diffusion theory, developed for spherical piezometers (Gibson 1963) and cylindrical piezometers (Brand and Premchitt 1982).

The finite element mesh involved triangular elements. To check that the basic mesh was sufficiently refined, a number of key analyses were repeated using a substantially refined finite element mesh, and the results were found to be the same as those obtained with the basic mesh to within 1%. The lateral boundary was generally assumed to be 5 m from the centreline of the piezometer. To check that this choice had no effect on the results, key analyses were repeated with the boundary at 2.5 m from the axis of the piezometer, and the results were found to be the same as for the base case to an accuracy of within 1%. As a check on the time integration steps, the number of time steps was increased by 100% and the results were found to be within better than 1% of those for the base case.

Results of analyses

Selection of dimensionless variables

The piezometer response is associated with the consolidation of the surrounding soils, therefore the radial (T_r) and vertical (T_v) time factors are two possible dimensionless variables. For an isotropic soil these factors can be written as $T_r = C_3t/a^2$ and $T_v = C_3t/z^2$, respectively, where t is the elapsed time, z is the distance of the centroid of the piezometer from the aquifer, a is the radius of the piezometer, and C_3 is the coefficient of consolidation under 3D strain conditions (the reader not familiar with the difference between the 1D, 2D, and 3D coefficients of consolidation is referred to Davis and Polous (1972) for a detailed discussion).

Another key dimensionless variable is associated with the continuity of flow relationship at the piezometer–soil interface. Based on continuity of flow at the interface of cylindrical piezometers, Brand and Premchitt (1982) established a factor λ where

$$[10] \quad \lambda = \frac{0.5\pi d^2 l m_{v3}}{V}$$

and where d is the diameter, l is the length, and V is the volume factor of the piezometer. The coefficient of volume change of the soil is m_{v3}, and it can be written for 3D strain conditions as

$$[11] \quad m_{v3} = \frac{3(1-2\nu)}{E}$$

where E and ν are the Young's modulus and Poisson's ratio of soil skeleton, respectively.

As shown in [10], the factor λ is a function of the surface area ($0.5\pi d^2 l$) of the piezometer, flexibility of the measuring system (i.e., volume factor V) and the soil compressibility (m_{v3}).

The value of the coefficient of consolidation depends on the prevailing strain conditions (Davis and Poulos 1972). For 1D strain conditions, the coefficient of consolidation C_1 can be written as

$$[12] \quad C_1 = \frac{K'E}{\gamma_w} \frac{(1-\nu)}{(1-2\nu)(1+\nu)} = \frac{K'}{S_s'}$$

where K' is the hydraulic conductivity, and S_s' the specific storage. For 3D strain conditions the coefficient of consolidation C_3 can be written as

$$[13] \quad C_3 = \frac{K'E}{\gamma_w (1-2\nu)} = \frac{(1+\nu)}{3(1-\nu)} \frac{K'}{S_s'}$$

By eliminating E in [11] and [12], m_{v3} can be written as

$$[14] \quad m_{v3} = \frac{3S_s'}{\gamma_w} \frac{(1-\nu)}{(1+\nu)}$$

By substituting m_{v3} in [10] λ can be written as

$$[15] \quad \lambda = 1.5 l S_s' \frac{(1-\nu)}{(1+\nu)} \frac{\pi d^2}{A}$$

where A is the cross-sectional area of the riser pipe (the choice of l and d for situations where there is a sand pack around the piezometers will be discussed in a later section). An appropriate value for Poisson's ratio ν should be used in the calculation of λ. Poisson's ratio ν can be obtained from drained triaxial tests. Typical ranges of values of ν are 0.3–0.35 for silt, 0.3–0.4 for soft clay, and 0.2–0.3 for stiff clay (Lee et al. 1983).

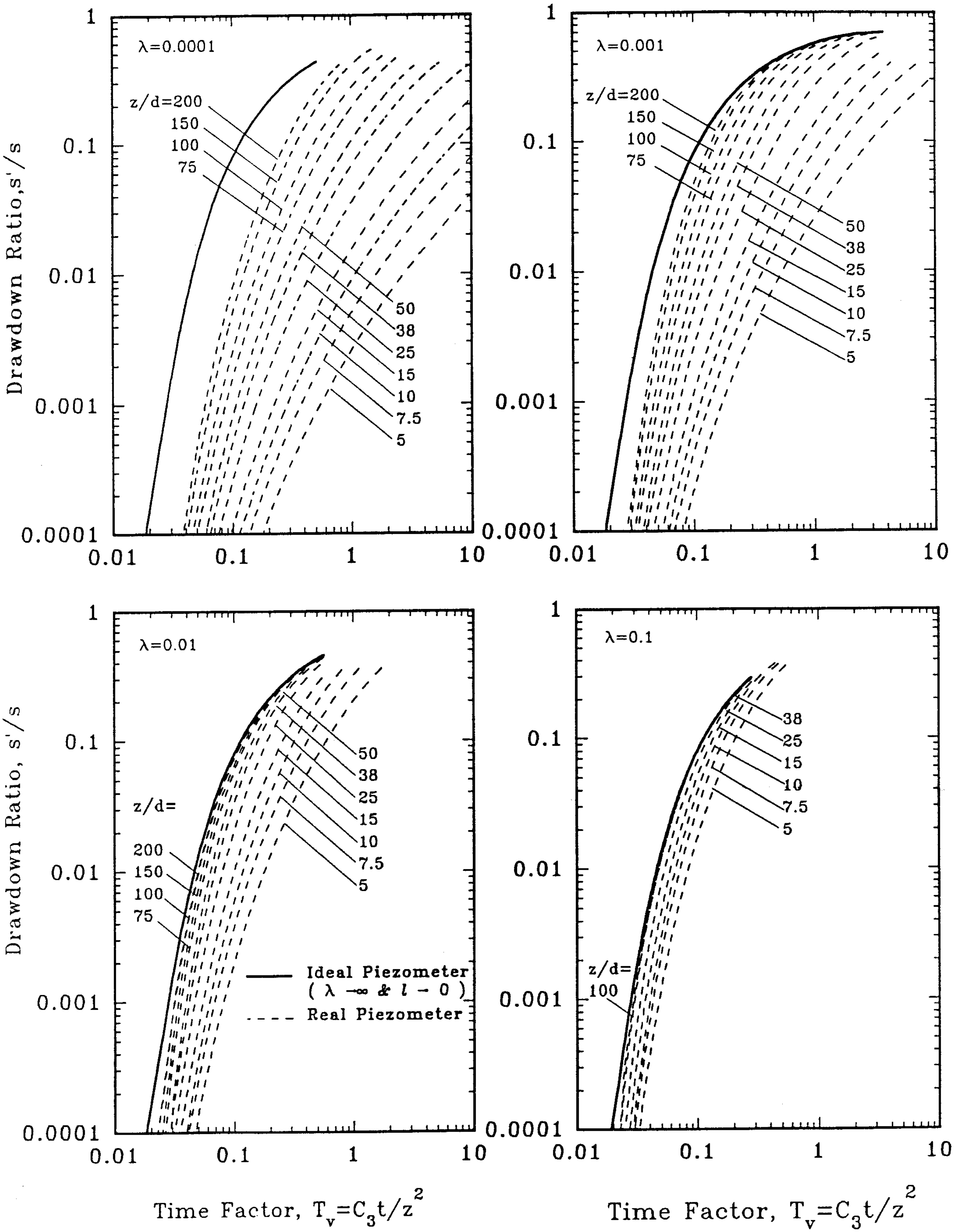

FIG. 4. Piezometer response curves: drawdown ratio vs. time factor.

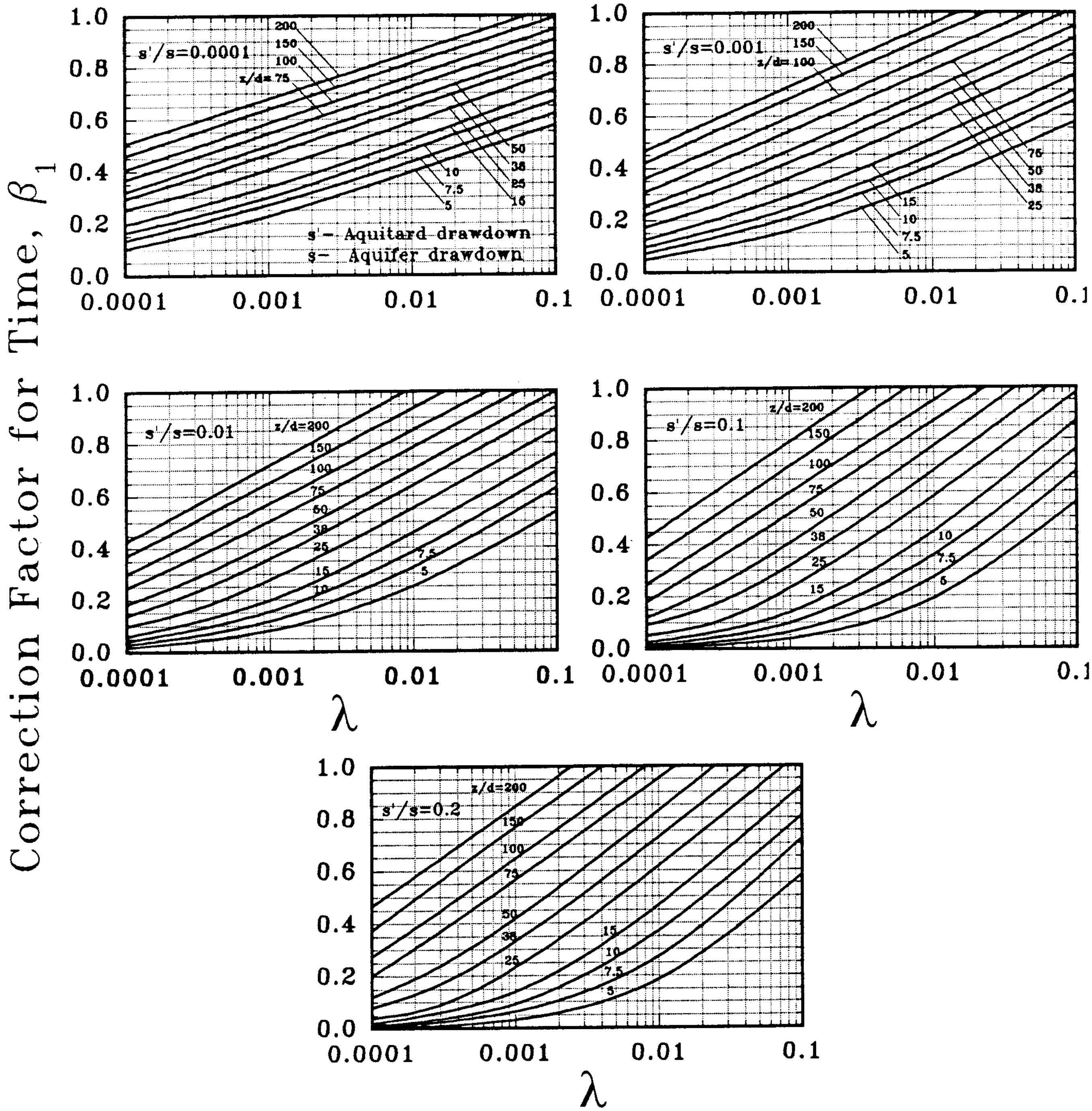

FIG. 5. Correction factor for time, β_1, vs. λ for drawdown ratios s'/s of 0.0001, 0.001, 0.01, 0.1, and 0.2.

The specific storage S_s' may be estimated based on the results of 1D consolidation tests. However, factors such as sample disturbance, gas in the soil, and the stress range can affect the laboratory results. The field value is likely to be less than or equal to the laboratory value obtained from loading tests. In many practical situations, the laboratory value of specific storage deduced from the loading and unloading stages of a consolidated test ay be taken as estimates of the upper and lower bound to the field specific storage (provided that representative samples and good experimental procedures were adopted). The effect of uncertainty regarding the specific storage can be assessed by repeating the estimate of hydraulic conductivity using reasonable upper and lower values for this parameter.

Piezometer response curves

The piezometer response curves in the form of drawdown ratio s'/s versus time factor T_v are shown in Fig. 4 for λ = 0.0001, 0.001, 0.01, and 0.1, where s' is the drawdown indicated by the piezometer in the aquitard, and s is the drawdown in the aquifer. The solid curve shown in each plot in Fig. 4 corresponds to the appropriate response curve of an "ideal" piezometer which has negligible length (i.e., $l \rightarrow 0$) and exhibits an immediate response to the pressure-head drop (i.e., there is no change in fluid volume; $A \rightarrow 0$, $\lambda \rightarrow \infty$). This is the same as the curve obtained by Neuman and Witherspoon (1972) for large t_D values (see Fig. 1) and is independent of the z/d ratio. The broken curves show the calculated drawdown for "real" piezometers, for various

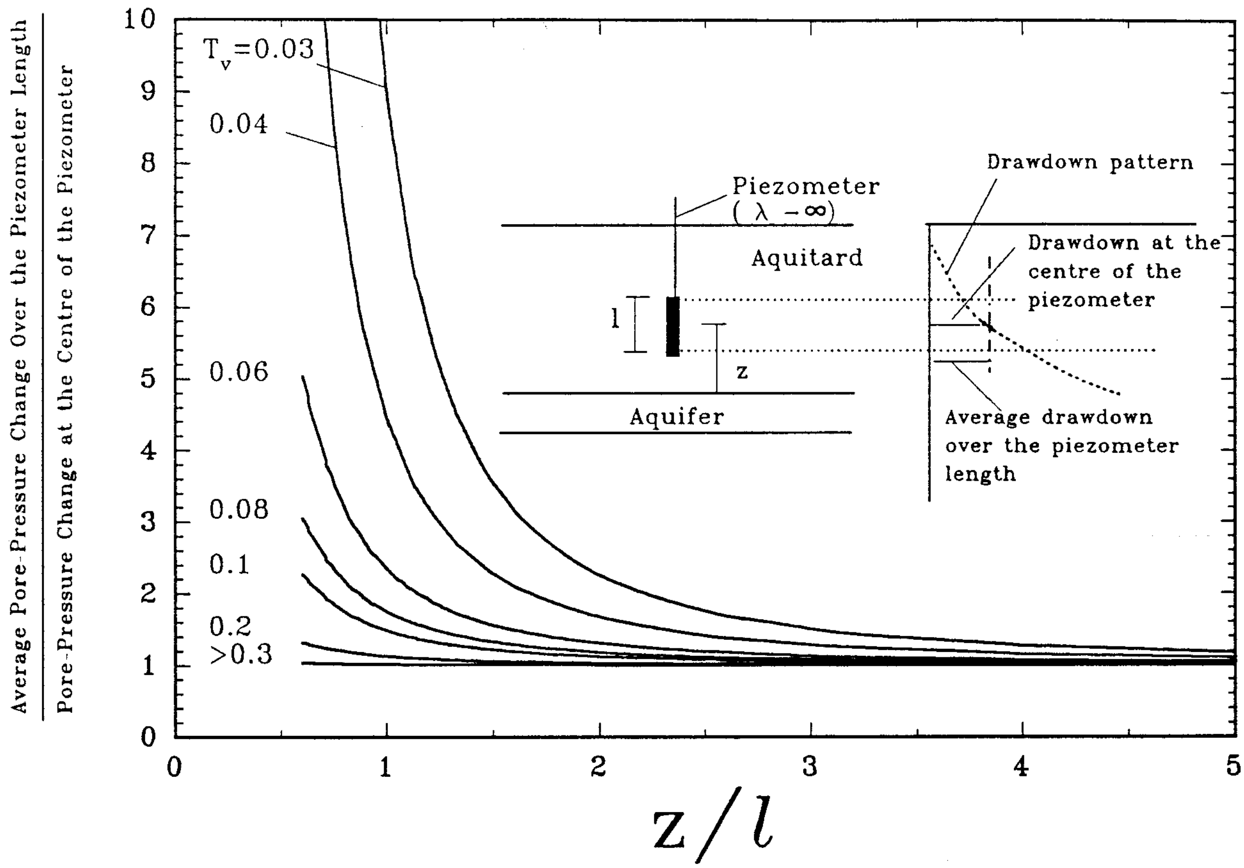

FIG. 6. Drawdown variation over the length of a piezometer ($\lambda \to \infty$). Average drawdown is much higher than that at the centre of the piezometer for $z/l < 4$ and when $T_v < 0.3$.

values of z/d in the range between 5 and 200, where z is the distance of the centre of the piezometer above the aquifer, and d is the diameter of the piezometer.

Time lag

The response curves in Fig. 4 indicate that there is a time lag between the response of the ideal and the real piezometers. The time lag is significant for lower values of λ and is very small for higher values of λ. Moreover, for a given value of λ the time lag decreases with increasing z/d values (i.e., the time lag is greatest, in relative terms, when the piezometer is located near the aquifer) because of the large change of gradient that occurs along the length of the piezometer as it approaches the aquifer, especially at early times.

Correction factor for time

Inspection of the response curves shown in Fig. 4 suggests that the response time of a piezometer can be corrected with reference to the ideal piezometer. A correction factor for time, β_l, is therefore defined herein as the ratio between the time taken by an ideal piezometer and the real piezometer to register a given drawdown. This ratio is the same as the ratio of the corresponding time factors (T_v). The correction factor β_l computed in this manner is shown in Fig. 5 as β_l versus λ for various drawdown ratios s'/s. This factor was established based on the commonly used value of Poisson's ratio of 0.3. The error in using these curves for other Poisson's ratios in the practical range of values, $\nu = 0.2$ to 0.4, is less than about ±10%.

Based on the foregoing, the use of Neuman and Witherspoon (1972) chart (Fig. 1) would be adjusted by introducing the correction factor β_l to account for the time lag of the piezometer; thus for a given ratio of s'/s, a dimensionless time t'_D can be evaluated from Fig. 1. However, in estimating the bulk hydraulic conductivity K', the time $t^* = \beta_l t$, where β_l is obtained from Fig. 5, and t is the actual time at which the drawdown ratio s'/s was observed, and t^* is the corresponding time that it would have taken an ideal piezometer to respond; thus

$$[16] \qquad K' = \frac{t'_D S'_s z^2}{t^*} = \frac{t'_D S'_s z^2}{\beta_l t}$$

For example, if the drawdown ratio (s'/s) of 0.001 is recorded by a piezometer at time $t = 1000$ min, and if $\lambda = 0.001$ and $z/d = 25$, the correction factor for time (β_l) would be about 0.35 (from Fig. 5), thus the aquitard response at 1000 min would correspond to the response of an ideal piezometer at time $t^* = 1000 \times 0.35 = 350$ min; thus in this example failure to consider time lag (i.e., if Fig. 1 were used without correction) would result in the hydraulic conductivity being underestimated by about a factor of three.

Effect of length and the location of piezometer

If the piezometer is located near the aquifer or if it is relatively long, a significant change in downward hydraulic gradient will occur over its length at early times. If the distance z to the centre of the piezometer is less than about four times the length of piezometer, l, the average drawdown over the length is found to be significantly higher than the drawdown of an ideal point piezometer located at the same elevation as the midpoint of the real piezometer. This suggests that the drawdown registered by the piezometer is

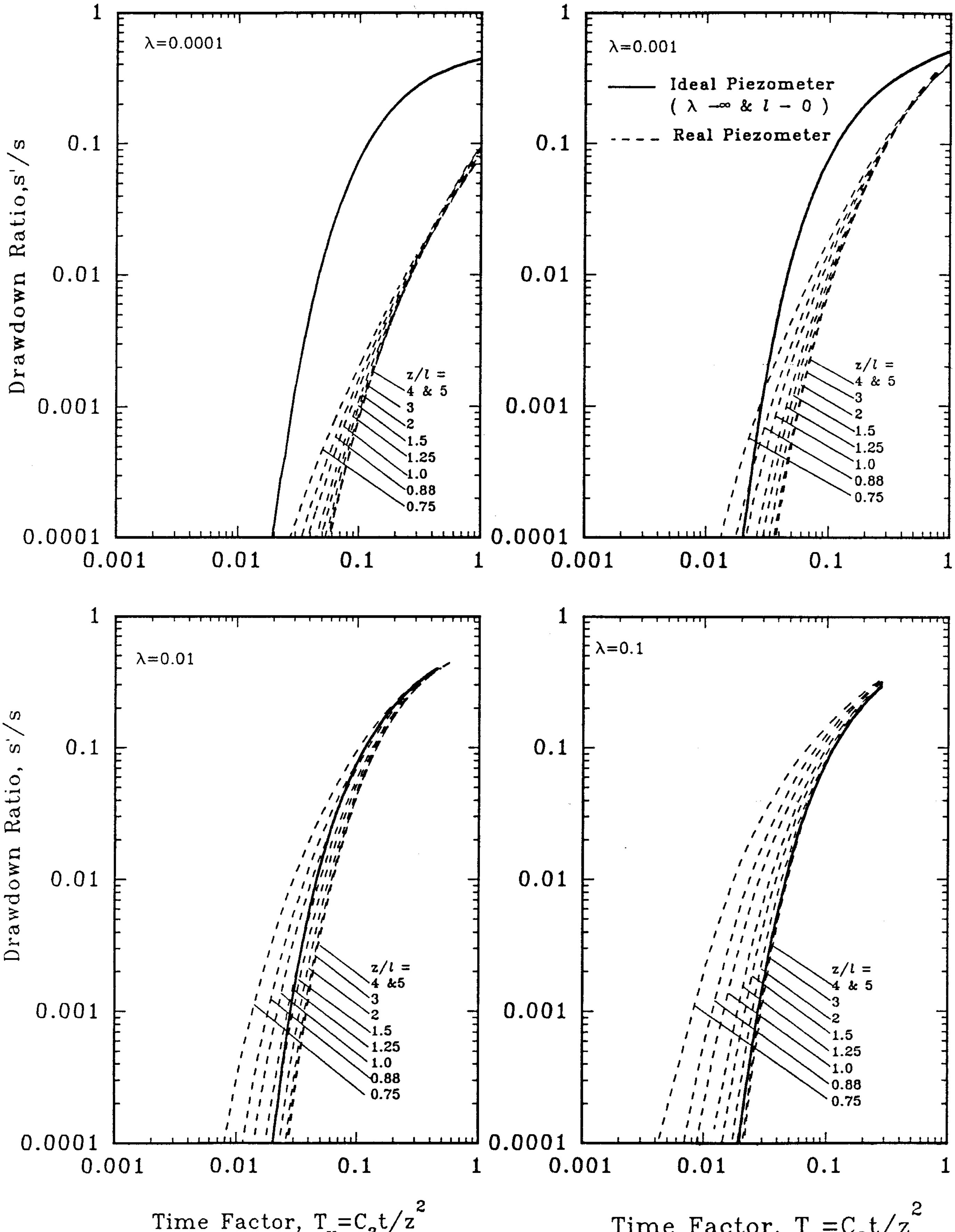

FIG. 7. Effect of piezometer length and its location relative to the aquifer (i.e., z/l) on the response curves.

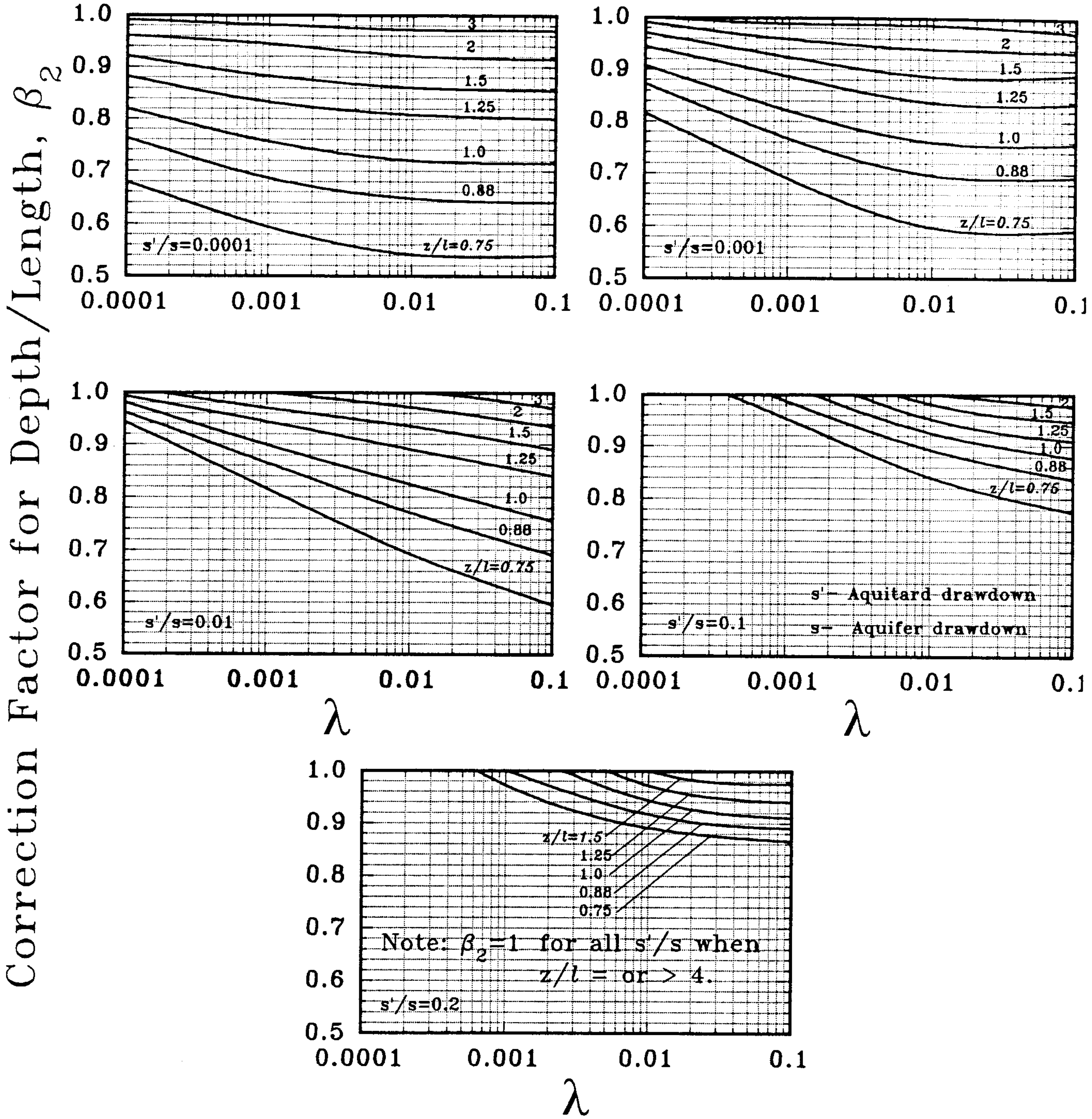

FIG. 8. Correction factor for depth/length, β_2, vs. λ for drawdown ratio s'/s of 0.0001, 0.001, 0.01, 0.1, and 0.2.

not representative of the actual pore-pressure change at the midlength of the piezometer but, rather, is representative of a point between the centre and bottom of the piezometer. This effect can be explained by considering the drawdown distribution over the length of a piezometer. Figure 6 shows the ratio of average pore-pressure change over the length of the piezometer to the pore-pressure change at the centre of the piezometer as a function of z/l for various times. This ratio may be substantially different from unity for z/l values less than about 4 at early times of drawdown. For larger times, when $T_v > 0.3$, the ratio is close to one. For situations where the ratio is significantly greater than one, the use of a distance z from the aquifer to the centroid of the piezometer as the value of z used in association with Neuman and Witherspoon (1972) charts would result in errors in the assessment of the bulk hydraulic conductivity of the soil.

Figure 7 shows the response of piezometers of finite length for a range of values of z/l between 1 and 5 for various λ values. It can be seen that the piezometer response curves for z/l ratios of 4 and 5 are almost identical and that for lower values the curves are shifted to the left. For low values of z/l, the curves may even be shifted left of the solid curve corresponding to the ideal piezometer, giving the impression that the simple piezometers can respond faster than an ideal piezometer located at the position of the centroid of a simple piezometer.

Low z/l values may represent either piezometers installed close to the aquifer or piezometers of relatively large length.

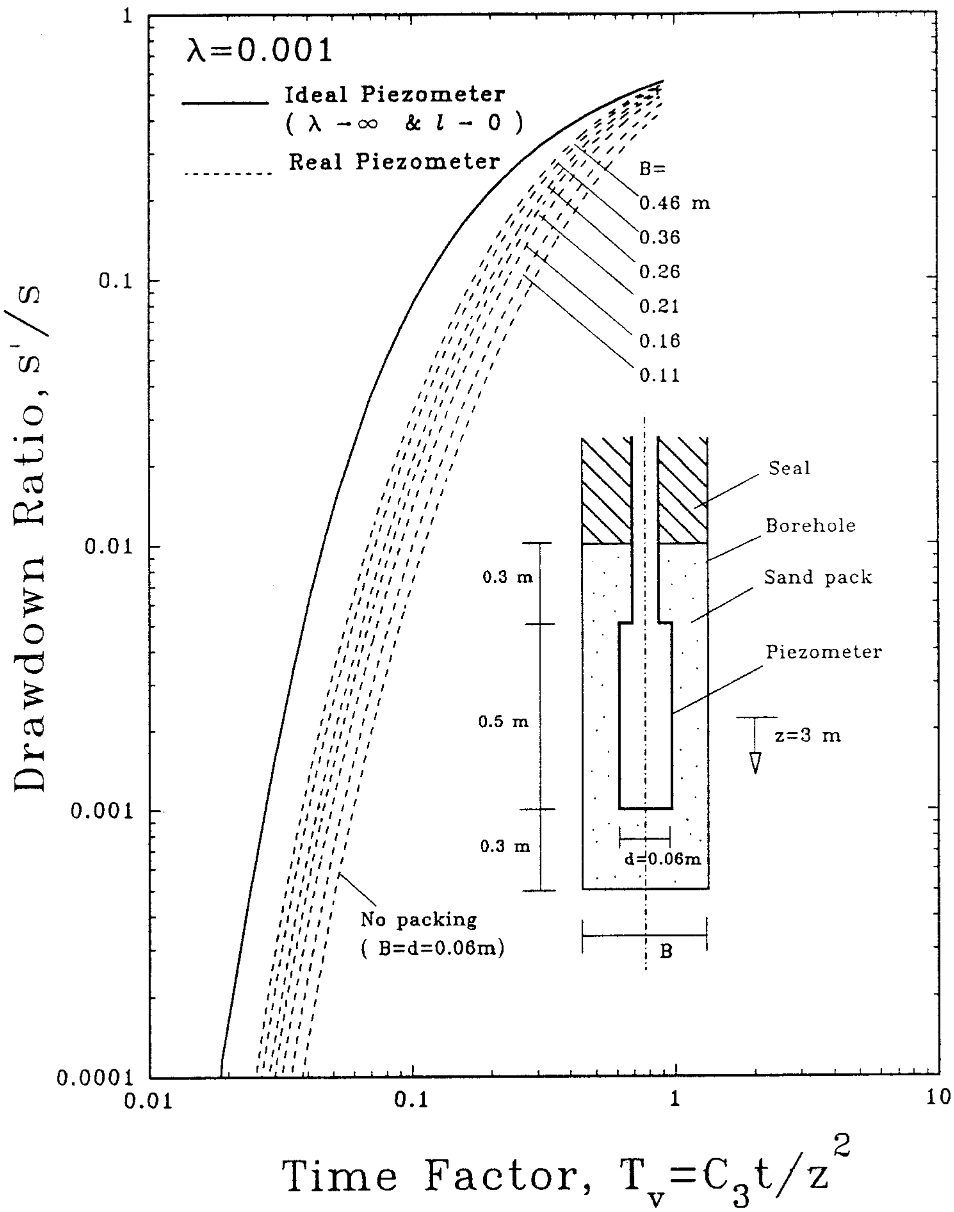

FIG. 9. Effect of borehole diameter B on response curves.

These shifts (called the depth/length effect) can be explained by virtue of the fact that the value of z used in the time factor T_v in Fig. 7 is to the centroid of the piezometer, although the actual response of the piezometer is not representative of the pore-pressure change at that elevation (as assumed for the ideal piezometer whose response is illustrated by the solid curves in Fig. 4 or by the Neuman and Witherspoon (1972) chart, see Fig. 1).

To allow the use of the Neuman and Witherspoon (1972) chart (Fig. 1) for values of $z/l < 4$ it is necessary to introduce a correction factor for the depth/length effect. A correction factor β_2 is defined herein as the ratio of the square root of the time factors for $z/l < 4$ to that for $z/l = 4$. The correction factors obtained on this basis are shown in Fig. 8 as a function of λ for various values of drawdown ratio s'/s. It should be noted that $\beta_2 = 1$ for all s'/s when $z/l \geq 4$.

For cases where the piezometer is installed closer to the aquifer or the length is relatively large so that the ratio $z/l < 4$, the solution obtained for an ideal piezometer (e.g., from the Neuman and Witherspoon (1972) chart, Fig. 1) may be corrected using the correction factors β_2 given in Fig. 8. Thus for a given value of s'/s a dimensionless time t'_D can be evaluated from Fig. 1; however, in estimating the bulk hydraulic conductivity K', the distance $z^* = \beta_2 z$ (where β_2 is obtained from Fig. 8, and z is the actual distance from the centroid of the piezometer to the aquifer) should be used. For example, if a piezometer of length $l = 1.5$ m is installed at a distance $z = 2.25$ m from the centre of the piezometer to the aquifer, the correction factor β_2 would be about 0.88 (from Fig. 8) for $\lambda = 0.01$ and $s'/s = 0.001$. This means that drawdown would be representative of a distance $z^* = 2.25 \times 0.88 = 1.98$ m instead of 2.25 m.

Effect of borehole size

The results reported so far were for a cylindrical piezometer installed in a borehole of the same diameter and length (i.e., without any gravel or sand pack). In practice, however, the borehole diameter is often greater than that of the piezometer screen, and a gravel or sand pack is often placed around the piezometer screen. It is of interest to consider

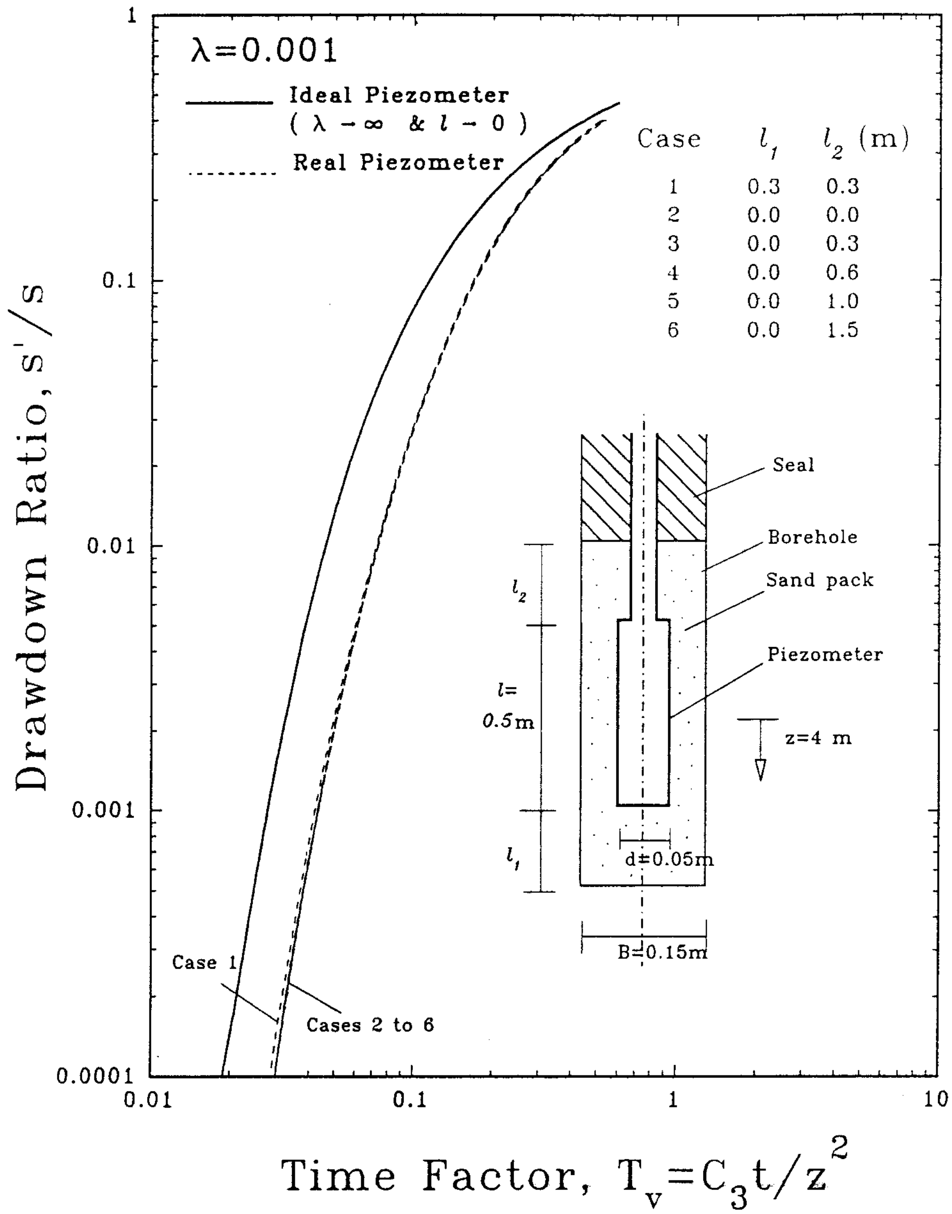

FIG. 10. Effect of extending the sand pack (above and below the piezometer) on the response curves. l_1 and l_2, length of sand pack below and above the piezometer screen, respectively.

how the borehole diameter and the sand–gravel pack affect the response.

Figure 9 shows the theoretical response of a piezometer of fixed diameter for various borehole diameters B (assuming a sand–gravel pack around the piezometer). It can be seen in Fig. 9 that the increase in borehole size enhances the response of a piezometer (i.e., decreases the time lag). This conclusion is based on the assumption that the sand–gravel packed around the piezometer is saturated. However, in practice, the larger the sand–gravel pack, the greater is the difficulty of deairing the piezometer and the greater the likelihood of entrapped air in the sand–gravel pack which can increase the time lag. Thus, although theory suggests that a larger borehole is beneficial, practical consideration would suggest that the smaller the diameter of the borehole (and consequently the smaller the diameter of the sand–gravel pack), the smaller is the likelihood of entrapped air affecting the results (all other things being equal) and hence, potentially, the better the results.

Effects of the length of sand pack

In practice, the sand pack often extends below the bottom and above the top level of the piezometer screen. To investigate the effect of this sand pack, the response of a piezometer was studied by varying the length of the pack below and above the piezometer screen, and the results are shown in Fig. 10. In case 1, the piezometer had 0.3 m pack above and below the piezometer screen. In other cases (cases 2–6) no packing was considered below the bottom of the piezometer screen but the length of the packing above the top of the screen was varied from 0 to 1.5 m. It can be seen in Fig. 10 that the response curves are identical for all the cases except for case 1 where the response is slightly earlier. The early response in case 1 is due to extending the pack below the bottom of the piezometer, thereby reducing the pathway between the piezometer and the aquifer. On the other hand, cases 2–6, which show an almost identical response curve, suggest that the sand pack above the top of the piezometer does not significantly influence the response

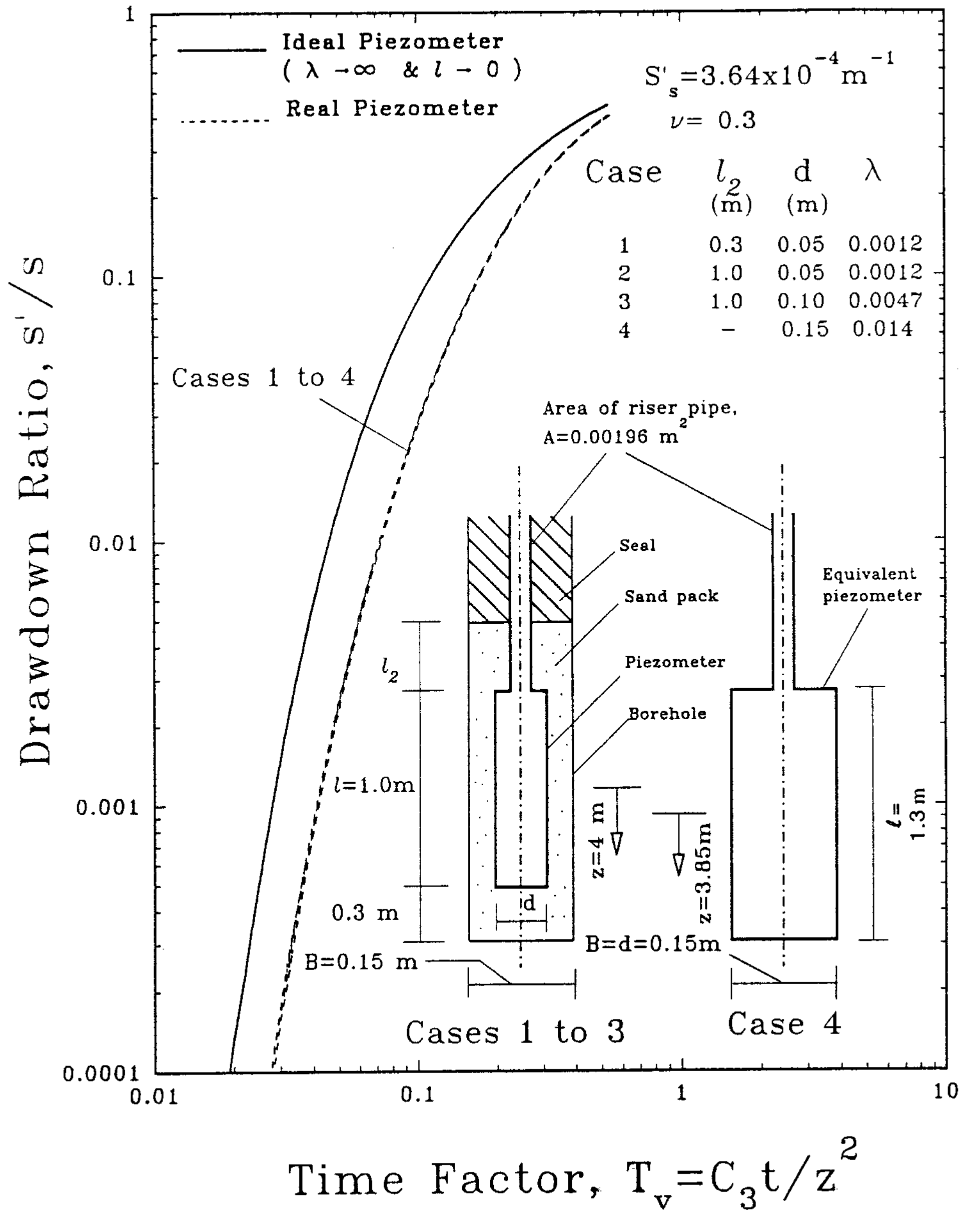

FIG. 11. Comparison of the response of piezometers with different sand-pack configurations to an equivalent piezometer without any sand pack. (*a*) Cases 1–4; (l = 1.3 m).

of the piezometer if z is measured to the centre of the piezometer screen.

Piezometer dimensions

Since piezometers are often installed in boreholes with a sand pack, it is customary to take the diameter of the borehole as the diameter of the piezometer (Fetter 1988; Penman 1960). The correction factors β_1 and β_2 (shown in Figs. 5 and 8, respectively), which were developed without any consideration of a sand pack, are equally applicable to the piezometers with a sand pack, provided that the diameter of the piezometer, d, is taken as the diameter of the borehole. To illustrate this, the response of a piezometer with a sand pack is compared (in Fig. 11) with an equivalent piezometer (with no sand pack) of diameter equal to the diameter of the borehole.

Cases 1 and 2 in Fig. 11*a* represent piezometers of 0.05 m diameter with a sand pack extending 0.3 and 1 m above the top level of the piezometer, respectively; case 3 represents a piezometer of 0.1 m diameter with 1 m packing above the top level of the piezometer. In all three cases, a 0.3 m sand pack was assumed below the bottom level of the piezometer, and the borehole diameter was taken as 0.15 m. The response of the piezometers with the configurations in cases 1–3 is essentially identical to that of the equivalent piezometer examined in case 4. The equivalent piezometer has a diameter of 0.15 m and an effective length of 1.3 m, thus the length of the packing above the top level of the piezometer screen is excluded in calculating the effective length.

The piezometer configurations considered in Fig. 11*b* are similar to that in Fig. 11*a* except for the borehole diameter. In Fig. 11*b* the borehole diameter is taken as 0.20 m. It can be seen that the response of the piezometer with configurations examined in cases 5–7 is identical to that of the equivalent piezometer, case 8. Thus it can be concluded

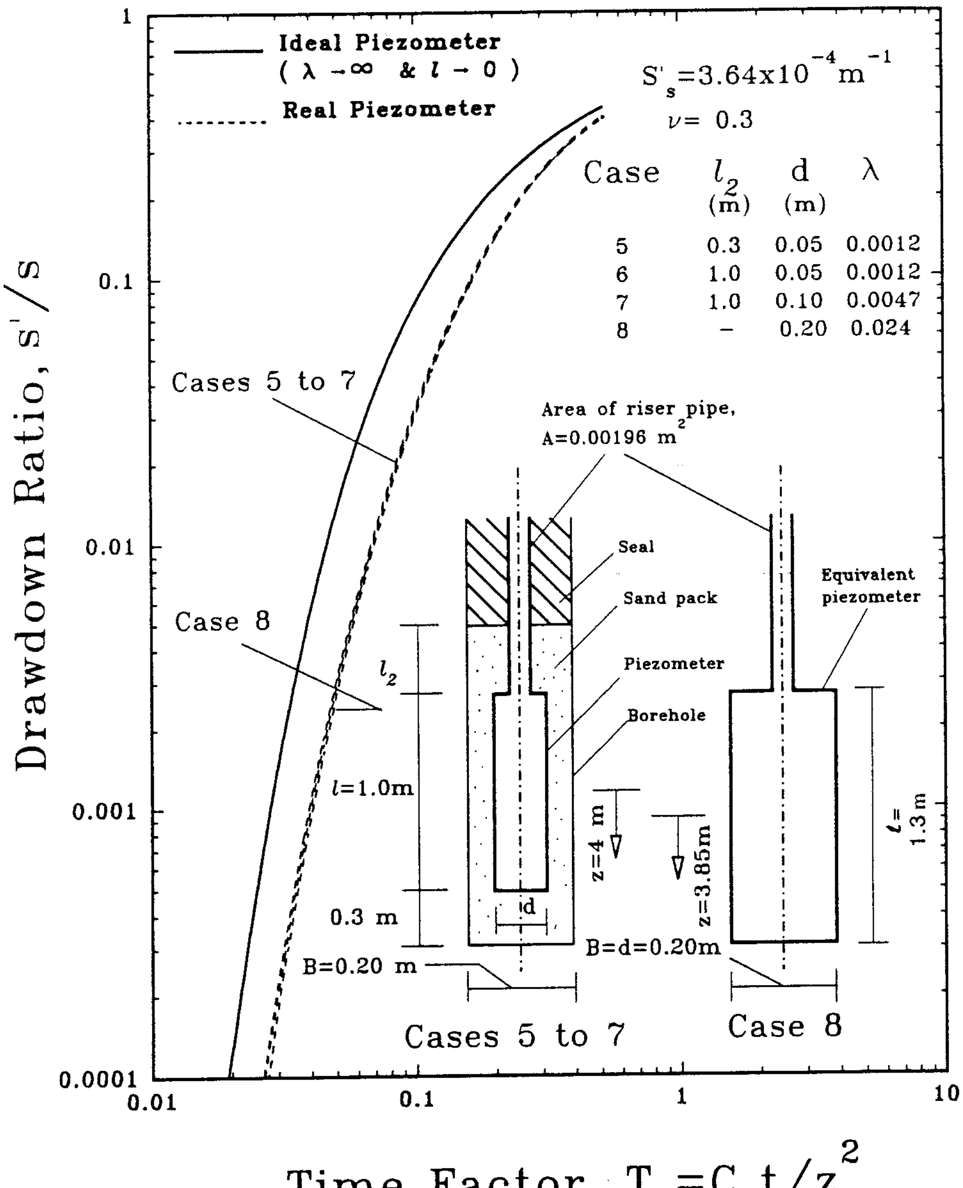

FIG. 11 (*continued*). (*b*) Cases 5–8; l = 1.3 m.

that, in the case of a piezometer with a sand pack, the correction factors β_1 and β_2 can be calculated using the borehole diameter and the effective length of the piezometer.

To illustrate further the effect of including the length of the top sand pack, the response curve for the piezometer with a sand pack (cases 1–3) is compared with that of the "equivalent" piezometer (cases 9 and 10) in Fig. 11*c*. The configurations of the piezometers in cases 1–3 are the same as those considered in Fig. 11*a*; however, in Fig. 11*c*, the length of the equivalent piezometer (cases 9 and 10) is taken to include both the bottom and the top sand pack. It is evident from the response curves that the equivalent piezometers obtained by including the full length of the sand pack are not appropriate. This further confirms that when calculating the length of the equivalent piezometer the top packing should be excluded; only the length of the screen and the sand pack close to the aquifer should be considered as in case 4 in Fig. 11*a*.

Application of correction factors

It has been shown that the simple piezometers used to monitor the drawdown in the aquitard as a result of a pump test on an adjacent aquifer may respond after a time lag which is related to both the characteristics of the piezometer and the soil. The representative depth of drawdown measurement will be shifted downward (at early times) for piezometers where $z/l < 4$. These two effects may be incorporated in the interpretation of the results of a pump test by means of correction factors applied to the conventional ratio method, as summarized below.

From a measured drawdown the hydraulic conductivity K' of the aquitard can be evaluated using Neuman and Witherspoon's (1972) curves to obtain t'_D as shown in Fig. 1:

$$[17] \quad K' = \frac{t'_D S'_s (z^*)^2}{t^*}$$

After making the corrections for time lag and the depth/length

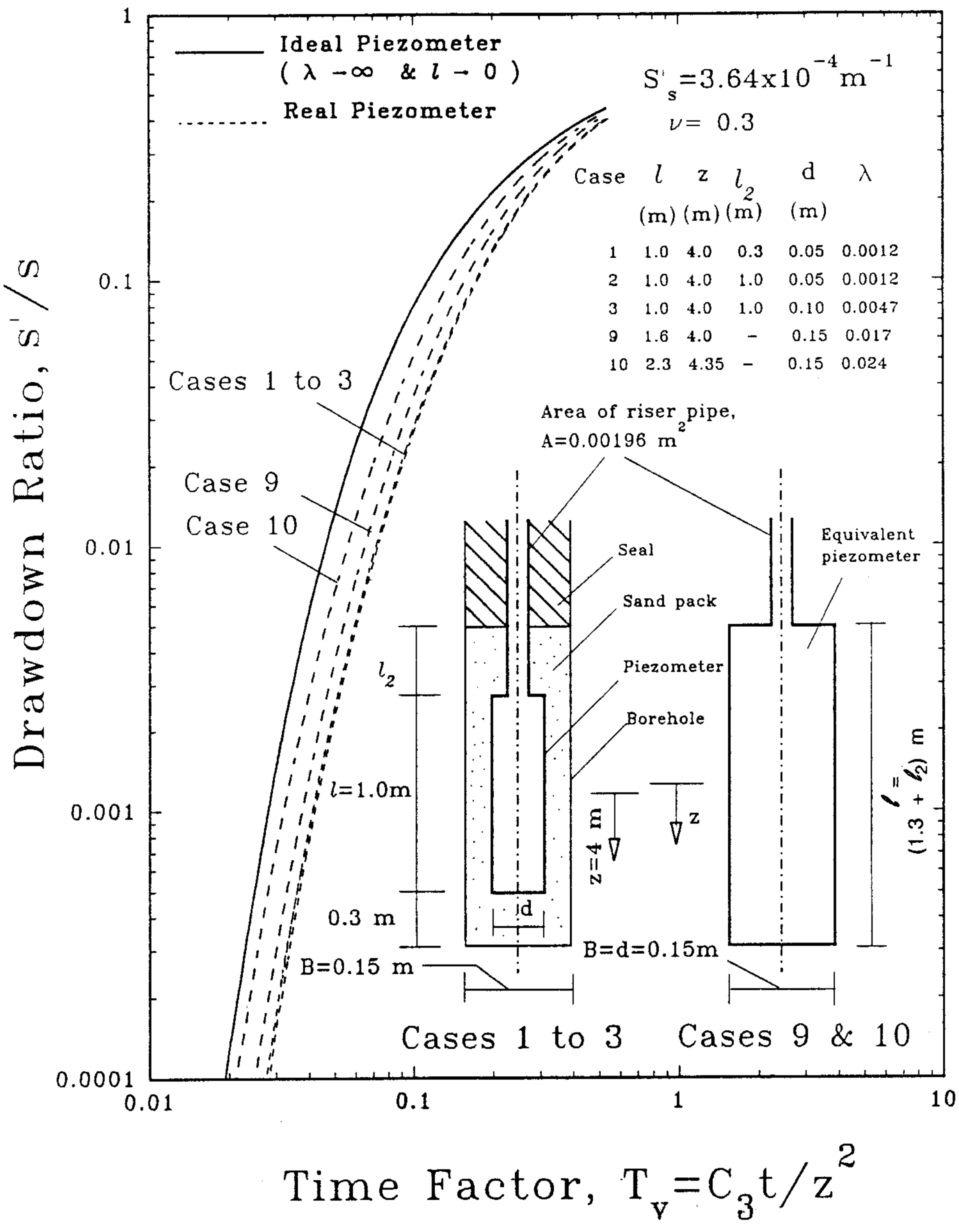

FIG. 11 (*concluded*). (*c*) Cases 1–3, 9, and 10; $l = (1.3 + l_2)$ m.

effect, [17*a*] can be written as

$$[18] \quad K' = \frac{t'_D S'_s (\beta_2 z)^2}{(\beta_1 t)} = \frac{t'_D S'_s z^2}{t} \frac{\beta_2^2}{\beta_1}$$

where t'_D is time factor, S'_s is the specific storage of the aquitard, z is the distance to the midpoint of the piezometer above the aquifer, t is elapsed time, and β_1 and β_2 are the correction factors for time (Fig. 5) and depth/length (Fig. 8), respectively. It should be noted, from [18], that β_2^2/β_1 is the gross correction factor that should be applied to the value K' that would be obtained directly from the ratio method (i.e., it reflects the "error" that would otherwise arise from not satisfying the ideal assumptions of Neuman and Witherspoon 1972).

To illustrate the application of the procedure, consideration will be given to adjusting the results from two hypothetical pump tests.

Example 1

Figure 12 shows the results from a pump test assumed to be conducted at a rate of 7.9 L/s, for 146 h, to help define the hydraulic conductivity of an aquitard. The drawdowns were monitored using observation wells which consist of a 50 mm diameter flush-joint PVC pipe, installed with a 1.5 m machine slot screen in a 0.15 m diameter borehole with a sand pack extending 0.5 m above the top level of the screen (no sand pack was placed below the bottom of the screen).

From Fig. 12 when t = 2710 min, s' = 0.019 m and s = 2.56 m, so the drawdown ratio becomes

$$\frac{s'}{s} = \frac{0.019}{2.56} = 0.0074$$

Using Jacob's method and the initial straight-line portion of the drawdown curve, the transmissivity T is estimated to be 7.87×10^{-4} m^2/s and the storage coefficient S is 1×10^{-5}. Thus for a monitoring nest located 574 m from the pump wells, [1*b*] gives

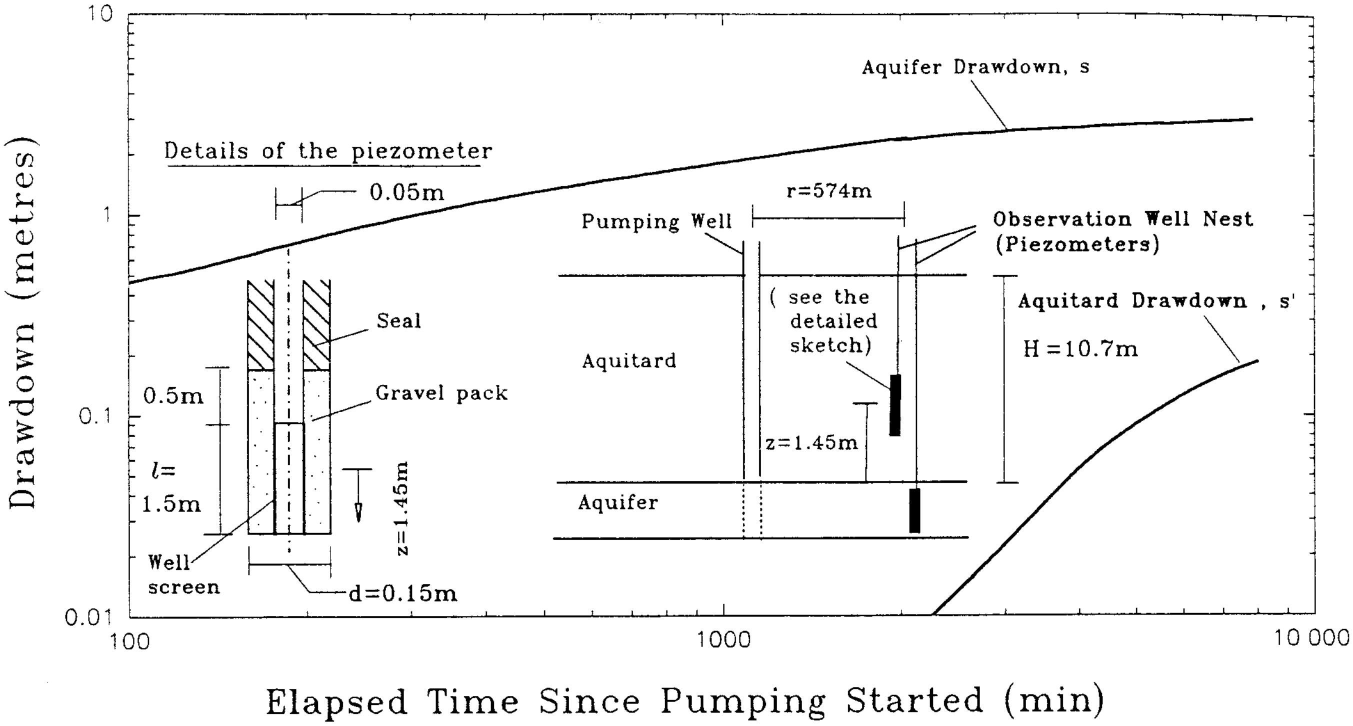

FIG. 12. Drawdown vs. elapsed time in a pump test. Example 1.

$$t_D = \frac{Tt}{Sr^2}$$

$$= \frac{(7.87\times10^{-4})(2710\times60)}{(1\times10^{-5})(574)^2} = 38.7$$

For the in situ stress range, an average specific storage of the aquitard obtained from consolidation tests is taken to be 1.5×10^{-3} m^{-1}. Assuming $\nu = 0.3$, λ can be calculated from [15]:

$$\lambda = 1.5 l S_s' \frac{(1-\nu)}{(1+\nu)} \frac{(\pi d^2)}{A}$$

$$\therefore \lambda = 1.5\times1.5(1.5\times10^{-3})\frac{1-0.3}{1+0.3}\times\frac{[\pi(0.15)^2]}{[\pi(0.05)^2]}$$

$$= 0.016$$

For $z = 1.45$ m, $d = 0.15$ m, and $l = 1.5$ m

$$\therefore \frac{z}{d} = \frac{1.45}{0.15} = 9.7, \qquad \frac{z}{l} = \frac{1.45}{1.5} \approx 1.0$$

From Fig. 5 the time correction factor β_1 is 0.46 (note that this value is not very sensitive to s'/s between the charts for 0.001 and 0.01), and from Fig. 8 the depth/length correction factor β_2 is 0.80 (interpolation is required between the chart for $s'/s = 0.001$ and 0.01). So the gross correction factor is

$$\frac{\beta_2^2}{\beta_1} = \frac{0.80^2}{0.46} = 1.4$$

It is noted that in this particular case the errors that would arise from neglecting the time lag and depth/length effect would be largely compensating and that hydraulic conductivity obtained from the normal application of the ratio method would underestimate the bulk hydraulic conductivity by 40%.

From Fig. 1, $t_D' = 0.082$ for $s'/s = 0.0074$. The hydraulic conductivity (in m/s) of the aquitard is therefore given by [18]:

$$K' = \frac{t_D' S_s' z^2}{t}\left(\frac{\beta_2^2}{\beta_1}\right)$$

$$= \frac{0.082\times1.5\times10^{-3}\times1.45^2}{2710\times60}\times1.4$$

$$\approx 2.2\times10^{-9} \text{ m/s}$$

Example 2

Figure 13 shows hypothetical pumping test data where an observation well (0.23 m diameter) in the aquitard was drilled to 28.96 m (95 ft). The 0.1 m diameter piezometer screen was assumed to be located (in the aquitard) from 28.65 to 28.96 m with a gravel pack from 28.35 to 28.96 m. The aquifer transmissivity T and the storage coefficient S were estimated independently to be 0.0184 m^2/s and 1.12×10^{-4}, respectively. From Fig. 13, when $t = 400$ min, $s' = 0.029$ m and $s = 3.66$ m, giving a drawdown ratio

$$\frac{s'}{s} = \frac{0.029}{3.66} = 0.0079$$

The magnitude of t_D at $r = 22$ m is given by [1*b*]:

$$t_D = \frac{Tt}{Sr^2} = \frac{0.0184(400\times60)}{(1.12\times10^{-4})(22)^2}$$

$$= 8.15\times10^3$$

From Fig. 1, $t_D' = 0.075$ for $s'/s = 0.0079$ and $t_D \approx 8 \times 10^3$. For $z = 3.2$ m, $l = 0.31$ m, $d = 0.23$ m, and $A = \pi(0.1)^2$

$$\frac{z}{d} = 13.91 \qquad \text{and} \qquad \frac{z}{l} = 10.3.$$

From 1D consolidation tests, the specific storage of the

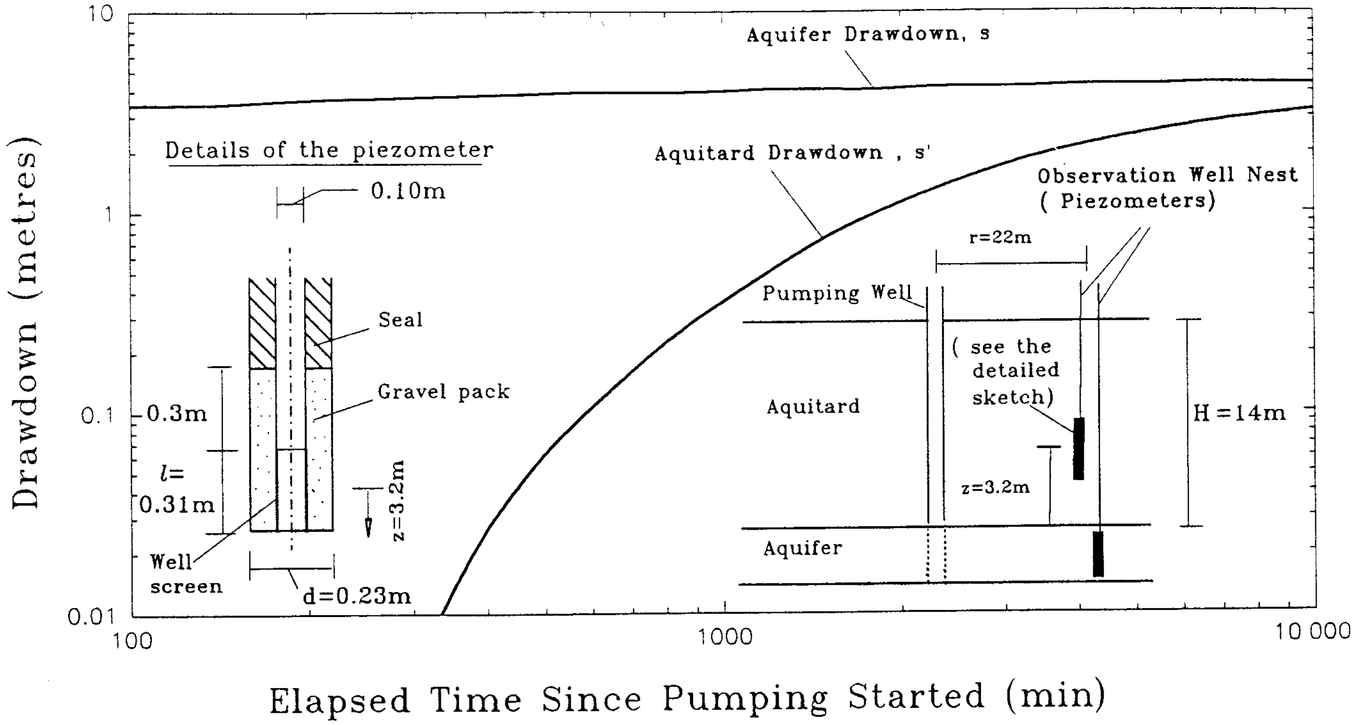

FIG. 13. Drawdown vs. elapsed time in a pump test. Example 2.

aquitard was $7.9 \times 10^{-4} \cdot m^{-1}$. Assuming $\nu = 0.3$, λ can be calculated from [15]:

$$\lambda = 1.5 l S_s' \frac{(1-\nu)}{(1+\nu)} \frac{(\pi d^2)}{A}$$

$$\therefore \lambda = 1.5 \times (0.31)(7.9 \times 10^{-4}) \frac{(1-0.3)}{(1+0.3)} \frac{\pi (0.23)^2}{\pi (0.1)^2}$$

$$= 0.001$$

From Fig. 5, $\beta_1 \approx 0.20$ and $\beta_2 = 1$, since $z/l > 4$.

The gross correction factor is then given by

$$\frac{\beta_2^2}{\beta_1} = \frac{(1.0)^2}{0.20} = 5$$

and hence neglecting time lag would result in an underestimate of the bulk hydraulic conductivity by a factor of 5, and the hydraulic conductivity (in m/s) of the aquitard is therefore given by [18]:

$$K' = \frac{t_D' S_s' z^2}{t} \frac{\beta_2^2}{\beta_1}$$

$$= \frac{0.075(7.9 \times 10^{-4})(3.2)^2}{400 \times 60} \times 5$$

$$= 1.26 \times 10^{-7} \text{ m/s}$$

Biot's theory versus diffusion theory

Diffusion theory has been widely used for the analysis of consolidation problems, largely because of its mathematical simplicity. The more rigorous Biot's theory (Biot 1941) has been less frequently used mainly because of its mathematical complexity. Both methods, however, give identical results when the Poisson's ratio of the soil skeleton (ν) is equal to 0.5 (i.e., when there is zero volume strain in the soil skeleton). Identical results also can be obtained for a 1D consolidation where the pressure change is constant (Gibson and Lumb 1953). While pumping an aquifer, the water flow in an adjacent aquitard would be essentially in the vertical direction, causing the aquitard to consolidate one dimensionally. The drawdown predicted at a point in the aquitard, therefore, would be identical for the 1D Biot and diffusion theories as long as the drawdown in the aquifer remains constant. However, in the vicinity of a real piezometer the water flow is essentially 3D: radial (axisymmetry) and vertical (downward). In this case the drawdown measured by the piezometer can be better described by Biot's theory than simple diffusion theory. To compare the two methods, the relationship between the hydraulic conductivity predicted by Biot's theory and diffusion theory, in the form of $K'_{\text{diffusion}}/K'_{\text{Biot}}$ versus λ, is established in Fig. 14 for various drawdown ratios s'/s, where $K'_{\text{diffusion}}$ is the hydraulic conductivity predicted by diffusion theory or from Biot theory assuming a Poisson's ratio of ~ 0.5, and K'_{Biot} is the hydraulic conductivity predicted by Biot's theory assuming a typical value of Poisson's ratio of 0.3.

It is evident from Fig. 14 that the diffusion theory would underpredict the hydraulic conductivity in comparison to Biot's theory. For higher λ values (say $\lambda > 0.01$) the difference is not significant, whereas for lower λ values (say $\lambda < 0.01$), which correspond to more compressible aquitards, the difference is up to 25%. More specifically, the hydraulic conductivity predicted by diffusion theory would be about 92–97% and 75–87% of that of realistic Biot's theory for higher and lower values of λ, respectively. This effect is automatically considered when the correction factors β_1 and β_2 are used as previously described.

Anisotropic aquitard

Aquitards are often deposited in more or less horizontal layers, causing the horizontal hydraulic conductivity to be

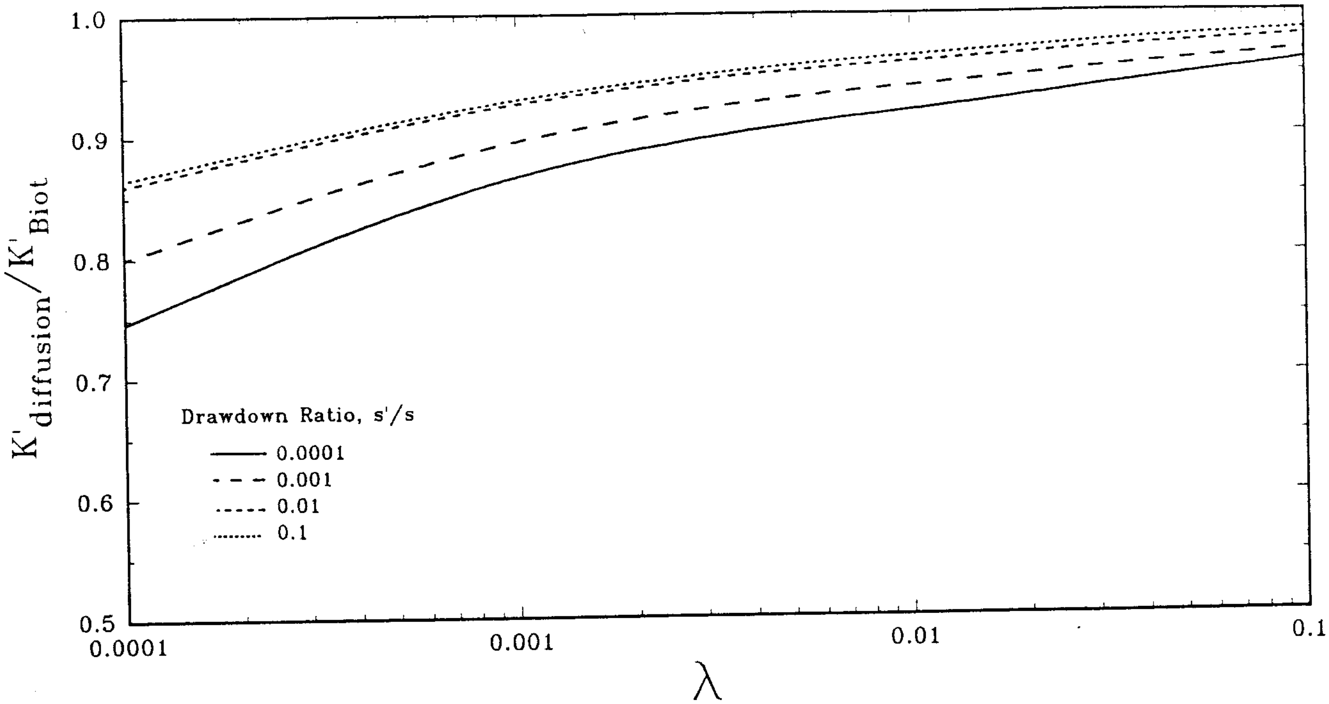

FIG. 14. Relationship between hydraulic conductivity predicted by diffusion theory ($K'_{\text{diffusion}}$) and that predicted by Biot's theory (K'_{Biot}).

greater than the vertical hydraulic conductivity. It is relatively common for the horizontal hydraulic conductivity (K'_h) to be 2–20 times higher than the vertical hydraulic conductivity (K'_v). It is therefore of interest to consider an anisotropic aquitard and examine how the correction factors developed earlier in this paper can be used for an anisotropic soil.

Since the flow in the aquitard is essentially 1D vertically downward, during a pump test, anisotropy does not have any effect on the response of an ideal piezometer. Neuman and Witherspoon (1972) demonstrated this using an anisotropic ratio as high as 250. However, for simple piezometers anisotropy does have some effect, as the water in the piezometer drains out horizontally, from the vertical face of the piezometer, into the surrounding soil when drawdown occurs. This means a simple piezometer will respond more quickly when the aquitard is anisotropic ($K'_h/K'_v > 1$) and that the response time will decrease with increasing anisotropy ratio (K'_h/K'_v).

To incorporate anisotropy in the analysis, the fluid stiffness matrix ϕ in [2*b*], [3], and [4] needs to be modified to include both horizontal and vertical hydraulic conductivities. Also, the horizontal hydraulic conductivity is used in the boundary-condition relationship (eq. [7]) at the piezometer–soil interface. The latter change requires the definition for λ be modified to account for anisotropy.

The λ used so far in this paper is similar to the one proposed by Brand and Premchitt (1982) for cylindrical piezometers used in other geotechnical applications for isotropic soils. It can be shown that λ for anisotropic aquitards should be modified as

$$[19] \qquad \lambda = 1.5\frac{K'_h}{K'_v} l S'_s \frac{(1-\nu)}{(1+\nu)}\frac{\pi d^2}{A}$$

where K'_h and K'_v are the horizontal (or radial) and vertical hydraulic conductivities, respectively, and all other terms are as defined for [15].

Using λ defined by [19], the correction factors β_1 and β_2 can still be obtained from the charts in Figs. 5 and 8, respectively. The error in using these charts, which were developed for isotropic aquitards, was found to be within 15 and 10% for β_1 and β_2, respectively, for anisotropic aquitards with K'_h/K'_v less than or equal to 20. The error will be a minimum for early drawdown ratios.

Summary and conclusions

The response of piezometers, installed in an aquitard, to a pumping test on an adjacent aquifer has been studied using a finite-element technique based on Biot theory. Attention has been focussed on a saturated piezometer installed in an isotropic aquitard for the case where the drawdown in the aquifer is rapid compared with the drawdown in the aquitard piezometer. This corresponds to large t_D in the conventional Neuman and Witherspoon (1972) approach. The solution can be approximately used for other cases involving an anistropic aquitard and small values of t_D. It has been demonstrated that piezometers commonly used in pumping tests may experience a time lag in comparison to the drawdown of an ideal piezometer which is assumed in the conventional method of interpretation. Correction factors to account for this time lag are presented. The delay is shown to be a function of the geometry and the flexibility of the piezometer and the compressibility of the surrounding soil (represented by a dimensional factor λ). The time lag is found to be significant for low values of λ and is negligible for high values of λ.

When the centre of the piezometer is located less than a distance of four times the length of the piezometer from the aquifer, the measured drawdowns may no longer be representative of the mid-depth of the piezometer. This occurs because of the large change in hydraulic gradient along the length of the piezometer. Correction factors are presented to allow the evaluation of a representative depth.

The correction factors can also be used for anisotropic aquitards with a small margin of error provided that the definition of λ is modified to incorporate the anisotropy ratio K'_h/K'_v.

Effect of borehole diameter and the presence of a sand pack were studied. It has been found theoretically that for a saturated piezometer and sand pack the time lag decreases with increasing diameter of the borehole. However, based on practical consideration of minimizing the potential volume of entrapped air, it is recommended that the diameter of the borehole (and hence the size of the sand–gravel pack) be kept as small as practical. It is shown that the sand pack above the top of the piezometer is theoretically insignificant and thus the effective length of the piezometer is the portion below the top level of the piezometer screen including the length of sand pack, if any, below the bottom level of the piezometer screen (where the pumped aquifer is assumed to be below the aquitard being monitored). From practical considerations, the length of sand pack above the screen should be kept as small as possible to minimize the potential volume of entrapped air. For piezometers with a sand pack, the borehole diameter and the effective length should be used as the piezometer dimensions when estimating the correction factors.

It is shown that diffusion theory may underpredict the hydraulic conductivity, as much as 25%, in comparison to more realistic Biot's theory. A correction for this effect is implicitly incorporated in the correction factors β_1 and β_2.

Acknowledgements

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