Two-dimensional pollutant migration in soils of finite depth

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A technique for analyzing two-dimensional migration of contaminant from a landfill into a homogeneous clayey layer is described. The analysis takes acount of the fact that the concentration of contaminant in the landfill will decrease as contaminant is transported into the soil. The analysis also allows for advective–dispersive transport within a permeable stratum (aquifer), which underlies the homogeneous layer. Consideration is given to chemical retardation arising from sorption/desorption of contaminant in the clayey layer. Some of the more important effects arising from the use of a 2-D analysis are illustrated by means of a limited parametric study. It is shown that the diffusion of contaminant from the aquifer into the clayey layer will substantially reduce the concentrations of contaminant in the groundwater away from the landfill. It is also shown that there is a critical velocity in the aquifer at which the maximum concentration at a point occurs. At greater or lesser velocities, significantly smaller concentrations may result. Thus design for limiting velocities is not necessarily conservative. Finally, the results of the 2-D analysis are compared with those from a 1-D analysis and the applicability of 1-D solutions is discussed.

Key words: contaminant migration, analysis, soil, diffusion, advection, pollutant.

Une technique pour analyser la migration à deux dimensions de contaminant d'un enfouissement sanitaire dans une couche argileuse homogène est décrite. L'analyse prend en compte le fait que la concentration de contaminant dans l'enfouisssement sanitaire va diminuer à mesure que du contaminant est transporté dans le sol. L'analyse tient compte également de la caractéristique d'advection et de disperson du transport à l'intérieur du substratum perméable (aquifère) localisé sous la couche d'argile. L'on prend en considération le retard de l'évolution chimique résultant de la sorption/désorption du contaminant dans la couche argileuse. Quelques-uns des effets les plus importants découlant de l'utilisation d'une analyse à deux dimensions sont illustrés au moyen d'une analyse paramétrique limitée. Il est démontré que la diffusion du contaminant de l'aquifère dans la couche argileuse va réduire appréciablement les concentrations de contaminant dans l'eau souterraine loin du site d'enfouissement. Il est démontré également qu'il y a une vitesse critique dans l'aquifère résultant d'une concentration maximum en un point. Des vitesses soit plus élevées ou moins élevées que cette vitesse critique peuvent produire des concentrations qui sont plus faibles de façon significative. Il n'est donc pas nécessairement sécuritaire de considérer des vitesses limites. Finalement, les résultats de l'analyse à deux dimensions sont comparés avec ceux obtenus en une dimension et l'applicabilité des solutions à une dimension est discutée.

Mots clés: migration de contaminant, analyse, sol, diffusion, advection, polluant.

[Traduit par la revue]

Introduction

The leachate from landfills or industrial waste disposal sites frequently contains toxic substances. Some care is necessary in choosing the location of these landfills to prevent significant amounts of these toxic substances from being transported into the groundwater system. Many regulatory authorities now require the use of liners to control pollutant migration (Weston 1978) and this often involves separating the landfill from any underlying aquifer by a relatively impervious natural clay layer or a compacted clay liner.

The movement of pollutants through relatively impermeable soils is quite slow; however, it is conceivable that significant pollution might occur in the long term and so it is important that such disposal sites be designed to prevent the possible contamination of the groundwater system in both the short and the long term.

The main factors that govern contaminant migration are advection, diffusion, and chemical reaction. For many applications the advective transport is small compared with that due to diffusion (see e.g. Goodall and Quigley 1977; Crooks and Quigley 1984). This situation has been investigated using both analytic and numerical techniques (see Freeze and Cherry (1979) for the general background and Anderson (1979) for a detailed review). However, a number of important practical situations do not appear to have been considered in existing solutions.

More recently, Rowe and Booker (1985*a*) have developed a one-dimensional analysis that takes into account the decrease in the concentration of leachate in the landfill as leachate is transported into the clay and that allows for the possible presence of a more permeable underlying stratum (aquifer) beneath the clay deposit. This analysis has been extended by Rowe and Booker (1984) to include the effects of several different soil layers.

The one-dimensional analysis may be adequate provided the width of the landfill is large compared with the depth of the clay layer and transport is predominantly vertical. If, however, the plan dimensions of the landfill are comparable with the depth of the layer, both horizontal and vertical transport may occur.

This paper considers the case of single-contaminant transport from a landfill into a homogeneous clay layer of finite depth under 2-D conditions. The analysis takes account of the fact that the surface concentration of the contaminant will decrease as leachate is transported into the clay layer, while also allowing consideration of advective transport due to groundwater flow within a permeable stratum, which underlies the clay layer. Consideration is also given to chemical retardation arising from adsorption/desorption of the contaminant by the clay.

The effect of considering two-dimensional transport in conjunction with these boundary conditions is illustrated by means of a limited parametric study. The range of situations wherein a simple 1-D analysis would be adequate will then be discussed.

Theory

Governing equations

The transport of substances through a saturated clay can often be approximated by a Fickian-type law (see e.g. Gillham and Cherry 1982), having the form

$$f_x = ncv_x - nD_{xx}\frac{\partial c}{\partial x}$$

[1]

$$f_z = ncv_z - nD_{zz}\frac{\partial c}{\partial z}$$

where n is the porosity of the soil layer; D_{xx}, D_{zz} are the coefficients of hydrodynamic dispersion (incorporating the effects of molecular diffusion and mechanical dispersion) in the x- and z-direction respectively; v_x, v_z are the horizontal and vertical component of the seepage velocity; $f_x = f_x(x,z,t) =$ the flux in the x-direction; and $f_z = f_z(x,z,t) =$ the flux in the z-direction.

Consideration of mass balance shows that

[2] $$\frac{\partial f_x}{\partial x} + \frac{\partial f_z}{\partial z} + n\frac{\partial c}{\partial t} + g = 0$$

where the quantity g takes account of the possibility of some of the contaminant being adsorbed onto the clay skeleton. For equilibrium-controlled ion exchange where the concentration of the exchange ion is relatively low, the adsorption of this species may be approximated by a linear relationship of the form

$$g = \rho K\frac{\partial c}{\partial t}$$

where ρ is the bulk density of solid and K is the distribution coefficient. The distribution coefficient K may often be estimated from the results of a laboratory column test (Rowe *et al.* 1985*b*) or may be determined independently (Griffin *et al.* 1976*a*, *b*). It should, of course, be determined over a representative range of concentrations reflecting the likely field variation.

For a soil in which the pore-fluid velocity is uniform, [1] and [2] can be combined to give

[3] $$\frac{\partial}{\partial x}\left(nD_{xx}\frac{\partial c}{\partial z}\right) + \frac{\partial}{\partial z}\left(nD_{zz}\frac{\partial c}{\partial z}\right) - \frac{\partial}{\partial x}(nv_x c)$$

$$- \frac{\partial}{\partial z}(nv_z c) = (n + \rho K)\frac{\partial c}{\partial t}$$

Problem configuration and boundary conditions

The problem under consideration is illustrated schematically in Fig. 1, which shows a typical section through a landfill of width L overlying a uniform clay layer of thickness H. Thus the following may be adopted: $c = c(x,z,t)$ is the concentration at any point (x,z) within the clay layer ($0 \le z \le H$) at time t; $c_T = c(x,0,t)$ is the concentration of the top of the clay layer ($z = 0$); $c_b = c(x,H,t)$ is the concentration in the aquifer beneath the clay layer ($z = H$); c_{LF} is the concentration of contaminant within the landfill.

It will be assumed that the concentration of the contaminant in the landfill c_{LF} is spatially homogeneous but may vary with time as leachate is transported into the clay; outside the landfill the surface concentration of contaminant will be assumed to be zero so that at the interface of landfill and clay ($z = 0$)

[4]
$$c_T = c_{LF}(t), \qquad |x| < L/2$$
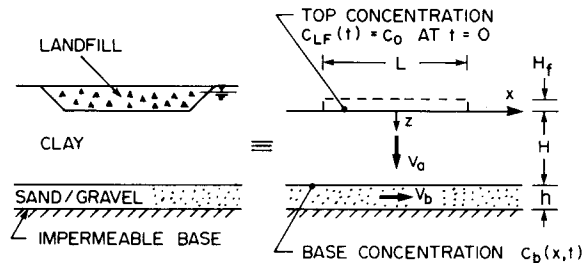$$c_T = 0, \qquad |x| > L/2$$

FIG. 1. Typical section through a landfill and problem idealization (not to scale).

The landfill will, of course, be placed over a period of time; however, this period will usually be small compared with the period of time necessary for a significant amount of contaminant to pass through the clay layer and so it may be assumed to sufficient accuracy that the placement of the landfill occurs instantaneously at $t = 0$. The initial conditions in the landfill can be specified in terms of the initial concentration $c_{LF}(0) = c_0$ and the volume of leachate. The volume of leachate can be expressed in terms of the equivalent height of leachate, H_f, which is defined as the volume of leachate per unit plan area of the landfill. The equivalent height of leachate (H_f) will depend on the porosity of the landfill, the position of the water table, and the degree of saturation. In practice, none of these quantities can be precisely defined; however, a reasonable estimate of all quantities can usually be made. The height of leachate will typically vary somewhat due to seasonal fluctuations, but since these changes are rapid in relation to the time scale of the problem, it seems reasonable to use an average value of H_f.

Consideration of conservation of mass within the landfill gives

[5a] $$LH_f c_{LF}(t) = LH_f c_0 - \int_0^t \int_{-L/2}^{L/2} f_z(x,0,\tau)dxd\tau$$

which simply states that the mass of contaminant in the landfill at time t is equal to the initial mass of contaminant ($LH_f c_0$) at $t = 0$ less the mass transported into the soil over the width of the landfill ($-L/2 \le x \le L/2$) between time $\tau = 0$ and time $\tau = t$. Equation [5a] may be rewritten as

[5b] $$c_{LF}(t) = c_0 - \frac{1}{LH_f}\int_0^t \int_{-L/2}^{L/2} f_z(x,0,\tau)dxd\tau$$

Now suppose that the clay layer is underlain by a more permeable stratum of thickness h and porosity n_b. If there is vertical advective transport into the aquifer, then, strictly speaking, continuity requires that the base velocity in the aquifer should vary with horizontal position. However, if the vertical superficial velocity $v_a = nv_z$ in the clay layer is small compared with the horizontal superficial velocity in the aquifer, then, as a first approximation, the base velocity v_b may be assumed to be uniform and horizontal. For any particular situation, the effect of this assumption can be assessed by analyzing the problem for the two cases where v_b is the expected velocity at $x = -L/2$ and $x = L/2$.

If it also assumed that the concentration in the aquifer is uniform across its thickness, then consideration of conservation of mass in an element of the aquifer between x and $x + dx$ gives

[6a] $$c(x,H,t)hn_b dx = \int_0^t f_z(x,H,\tau)d\tau dx$$

$$- h\int_0^t [f_x(x + dx,H,\tau) - f_x(x,H,\tau)]d\tau$$

which follows from the observation that the mass of contaminant contained in this element at time t is equal to total mass transported into the element from the overlying clay less the total mass transported out of the element by horizontal advection and dispersion. Dividing throughout by $hn_b dx$, taking the limit as dx tends to zero, and assuming that mass transport is governed by Ficks law gives

[6b] $c_b = c(x,H,t)$

$$= \int_0^t \left[\frac{f_z(x,H,\tau)}{hn_b} - \frac{v_b}{n_b}\frac{\partial c(x,H,\tau)}{\partial x} + D_H \frac{\partial^2 c(x,H,\tau)}{\partial x^2} \right] d\tau$$

where D_H is the coefficient by hydrodynamic dispersion in the horizontal (x) direction in the permeable aquifer.

Solution

The transport process is governed by [1]–[6] together with the condition that the concentration is initially zero in the clay layer. Clearly, the process is a planar one and so the field quantities depend only on x, z, and t. Attention will be restricted to the case in which the seepage velocity in the clay layer is uniform and corresponds to a constant superficial velocity v_a in the vertical direction so that

[7] $v_x = 0; \qquad v_z = v_a/n$

Equations [1]–[6] can be simplified by the introduction of a Laplace transform:

[8] $(\bar{c}, \bar{f}_x, \bar{f}_z) = \int_0^\infty (c, f_x, f_z) e^{-st} dt$

and a Fourier transform:

[9a] $(C, F_x, F_z) = \frac{1}{2\pi} \int_{-\infty}^\infty (c, f_x, f_z) e^{-i\xi x} dx$

The Fourier integral theorem shows that this equation is equivalent to the representation

[9b] $(c, f_x, f_z) = \int_{-\infty}^\infty (C, F_x, F_z) e^{i\xi x} d\xi$

If the transforms ([8] and [9]) are applied to [1] and [3] it is found that, for a homogeneous soil,

[10a] $\bar{F}_x = -nD_{xx}i\xi\bar{C}$

[10b] $\bar{F}_z = v_a\bar{C} - nD_{zz}\frac{\partial \bar{C}}{\partial z}$

[11] $D_{zz}\frac{\partial^2 \bar{C}}{\partial z^2} - D_{xx}\xi^2\bar{C} - \frac{v_a}{n}\frac{\partial \bar{C}}{\partial z} = \left(1 + \frac{\rho K}{n}\right)s\bar{C}$

It is then a simple matter to show that

[12a] $\bar{C} = \bar{C}_T \frac{(e^{\alpha Z + \beta} - e^{\beta Z + \alpha})}{e^\beta - e^\alpha} + \bar{C}_b \frac{(e^{\beta Z} - e^{\alpha Z})}{e^\beta - e^\alpha}$

[12b] $\bar{C}_T = \frac{1}{2\pi}\int_{-\infty}^\infty \bar{c}_T e^{-i\xi x} dx$

[12c] $\bar{C}_b = \frac{1}{2\pi}\int_{-\infty}^\infty \bar{c}_b e^{-i\xi x} dx$

where $\bar{C}_T$, $\bar{C}_b$ denote the transformed concentrations at the top and bottom of the layer respectively, $Z = z/H$, and $\lambda = \alpha$, β are the roots of the quadratic equation

[13] $D_{zz}\lambda^2 - \frac{v_aH}{n}\lambda - \left(\xi^2 H^2 D_{xx} + \left(1 + \frac{\rho K}{n}\right)sH^2\right) = 0$

The transformed components of flux $\bar{F}_T$, $\bar{F}_b$ entering the top and bottom of the layer respectively can then be calculated from [10b] and [12] and it follows that

[14]

$$\bar{F}_T = \frac{nD_{zz}}{H}(S_{TT}\bar{C}_T + S_{Tb}\bar{C}_b)$$

$$-\bar{F}_b = \frac{nD_{zz}}{H}(S_{bT}\bar{C}_T + S_{bb}\bar{C}_b)$$

where $S_{TT} = (\beta e^\beta - \alpha e^\alpha)/(e^\beta - e^\alpha)$, $S_{Tb} = (\alpha - \beta)/(e^\beta - e^\alpha)$, $S_{bT} = e^{\alpha+\beta}(\alpha - \beta)/(e^\beta - e^\alpha)$, $S_{bb} = (\beta e^\alpha - \alpha e^\beta)/(e^\beta - e^\alpha)$.

If boundary condition [6b] is transformed it follows that

[15] $\bar{C}_b = \frac{1}{s}\left[\frac{\bar{F}_b}{hn_b} - \left(\frac{i\xi v_b}{n_b} + \xi^2 D_H\right)\bar{C}_b\right]$

so that using [14],

[16] $\bar{C}_b = -\Phi\bar{C}_T$

where

$$\Phi = \frac{\dfrac{nD_{zz}}{shHn_b}S_{bT}}{1 + \dfrac{nD_{zz}}{shHn_b}S_{bb} + \dfrac{i\xi v_b}{sn_b} + \dfrac{\xi^2 D_H}{s}}$$

Equations [12] and [16] can now be used to express the solution in terms of the unknown quantity $\bar{C}_T$. In particular, the flux, $\bar{f}_T$, entering the top surface of the clay layer is given (using [9], [14], and [16]) by

[17] $\bar{f}_T = \frac{nD_{zz}}{H}\int_{-\infty}^\infty \chi e^{i\xi x}\bar{C}_T d\xi$

where $\chi = S_{TT} - \Phi S_{Tb}$. Thus,

[18] $\frac{1}{L}\int_{-L/2}^{L/2} \bar{f}_T dx = \frac{nD_{zz}}{H}\int_{-\infty}^\infty \chi\bar{C}_T \frac{\sin(\xi L/2)}{\xi L/2} d\xi$

Now from [12b] and [4] we have

[19a] $\bar{C}_T = \frac{1}{2\pi}\int_{-\infty}^\infty \bar{c}_T e^{-i\xi x} dx = \frac{1}{2\pi}\int_{-L/2}^{L/2} \bar{c}_{LF} e^{-i\xi x} dx$

which, on performing the integration, gives

[19b] $\bar{C}_T = \frac{L}{2\pi}\bar{c}_{LF}\frac{\sin(\xi L/2)}{\xi L/2}$

Whereupon, taking the Laplace transform of boundary condition [5b] and using [18] and [19b], it follows that

[20a] $\bar{c}_{LF} = \frac{c_0/s}{1 + \dfrac{nD}{sH^2}\cdot\dfrac{H}{H_f}\cdot\Omega}$

where

[20b] $\Omega = \frac{L}{2\pi}\int_{-\infty}^\infty \chi\left[\frac{\sin(\xi L/2)}{\xi L/2}\right]^2 d\xi$

The initial concentraion c_0 is known and the integral occurring in [20b] can be evaluated using numerical quadrature, thus $\bar{c}_{LF}$ can be determined. This implies that $\bar{C}_T$ can be found (eq. [19]) and so $\bar{C}_b$ can be found (eq. [16]) and thus the transformed concentration can be found (eq. [12]). The Fourier transform can now be inverted by numerical quadrature using

[9*b*] while the Laplace transform can be inverted using an efficient algorithm developed by Talbot (1979) and so the solution is complete.

Results

The theory presented in the previous section is amenable to the analysis of contaminant migration profiles for a wide range of geometries and combinations of parameters. To illustrate some of the more important effects, consideration will be given to the hypothetical case of a 200 m wide landfill ($L = 200$ m, see Fig. 1) resting on a 2 m thick homogeneous clay stratum ($H = 2$ m) with porosity $n = 0.4$, underlain by a relatively permeable 1 m thick sand stratum ($h = 1$ m) with porosity $n_b = 0.3$. For the purposes of this example, it is assumed that mass transport in the clay is purely by diffusion, the soil is isotropic with respect to diffusion (i.e., $D_{zz} = D_{xx} = D = 0.01$ m²/a), and the advective velocity is zero ($v = 0$). (The theory does, of course, permit consideration of advection in the clay as well as anisotropic diffusion, if required.)

Unless otherwise stated, the height of leachate H_f is taken to be 1 m, it is assumed that there is no geochemical reaction ($\rho K = 0$), the horizontal superficial velocity in the sand is taken to be 1 m/a ($v_b = 1$ m/a), and it is assumed that there is negligible horizontal diffusion/dispersion in the sand ($D_H = 0$). The effects of varying the leachate height H_f and the clay thickness H are very similar to those observed in the 1-D case (Rowe and Booker 1985a) and will not be examined here. The effect of varying the other parameters (ρK, v_b, D_H) will be discussed subsequently.

Natural attenuation within the sand layer

Figure 2 shows the variation in the concentration in the sand layer (c_b) with lateral position, at four times ($t = 100, 300, 500,$ and 1000 years) after completion of the landfill. Except at very low times, the concentration increases approximately linearly with lateral position beneath the landfill and in all cases attains a maximum value at the "downstream" (in the sense of the direction of flow in the base strata) edge of the landfill.

Outside the landfill, the concentration decreases with increasing distance from the landfill. At small times this decrease is primarily due to a time lag; however, at larger times it is primarily due to diffusion of contaminant back into the adjacent clay. Thus there is a natural attenuation in the system that will ensure the maximum concentration reached at any point outside the boundaries of the landfill will never reach the maximum value at the end of the landfill.

The concentration of contaminant within the sandy aquifer beneath the clay can be reduced by increasing the thickness H of the clay. However, if this is not possible, the concentration can also be controlled by providing a buffer zone between the landfill and the areas where the groundwater is required to have a specified quality (i.e., the concentration of contaminant must not exceed specified levels).

To illustrate the effect of the attenuation in a buffering zone, Fig. 3 shows the variation in contaminant concentration in the sand with time at the edge of the landfill ($x = 100$ m) and at a point 300 m "downstream" from the landfill ($x = 400$ m) for two values of ρK. Considering firstly the curves for the case where there is no adsorption ($\rho K = 0$) it is seen that the contaminant concentration at both positions increases with time until a peak value is reached and then decreases for subsequent time. As might be expected, there is a time lag between the times at which the peak values are reached at $x = 100$ m and $x = 400$ m
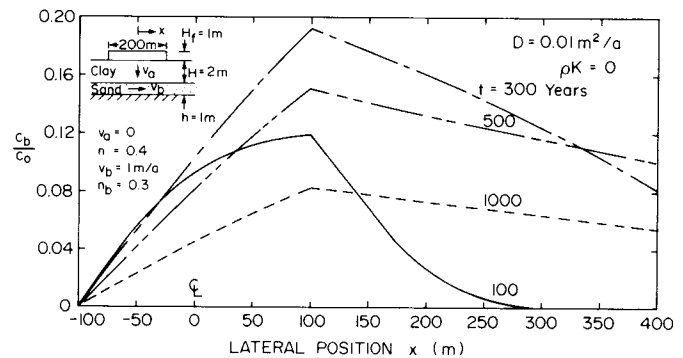

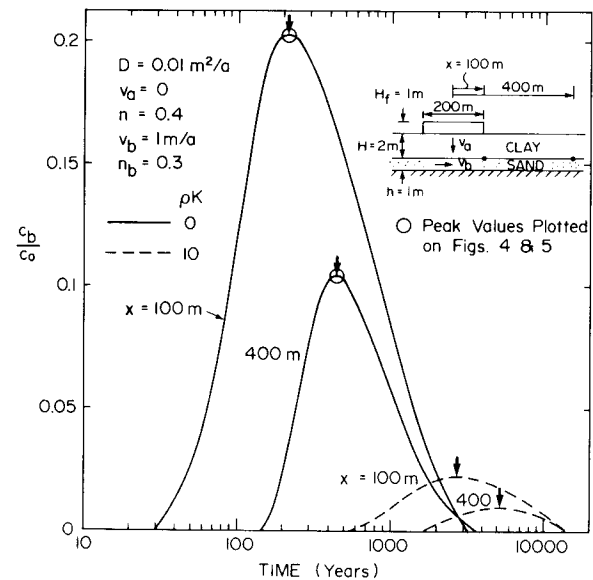FIG. 2. Variation in base concentration with position.

FIG. 3. Variation in base concentration with time.

(although because of the effect of diffusion into the clay this time lag does not correspond to what would be expected for piston flow in the sand). Of greater interest is the fact that the magnitude of the peak concentration is substantially reduced even for this analysis where no horizontal diffusion–dispersion in the sand layer is considered ($D_H = 0$).

Similar results were obtained for the case where there was moderate sorption of contaminant onto the clay (i.e., $\rho K = 10$) as shown in Fig. 3. As previously noted by Rowe and Booker (1985a) with regard to the 1-D case, sorption can significantly reduce the magnitude of the peak concentration that can be obtained in the sand layer as well as substantially increase the time required to reach the peak value. For a more detailed discussion of the effect of adsorption the reader is referred to Rowe and Booker (1985a, b).

Effect of advective velocity in the aquifer

The results presented so far have all been for a base velocity $v_b = 1$ m/a. Figure 4 shows the variation in the maximum base concentration c_b and the time required to reach this concentration at $x = 100$ m (i.e., the edge of the landfill) and $x = 400$ m for a range of base velocities. Beneath the landfill, the maximum concentration in the sand, c_b, decreases monotonically with increasing base velocity because of the higher dilution of contaminant that occurs for higher cross-flow velocities. However, outside the landfill two quite different mechanisms compete to control the maximum base concentra-
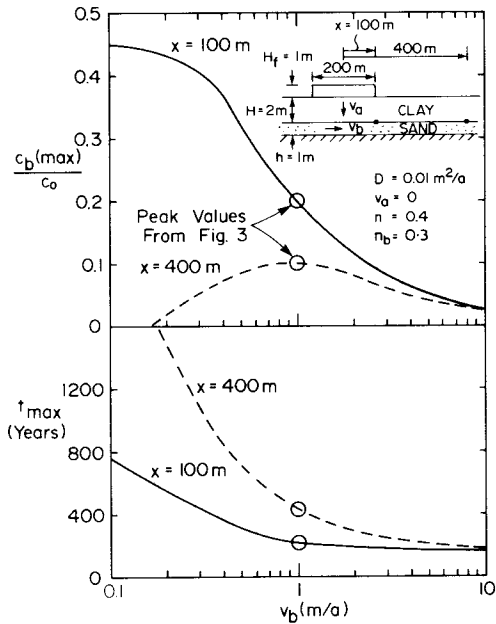
FIG. 4. Variation in maximum base concentration with base flow velocity.

FIG. 5. Variation in maximum base concentration with the horizontal dispersion coefficient in the base layer.

tion at a point. For points away from the landfill, the concentration is reduced (compared with the maximum value beneath the landfill) owing to diffusion into the adjacent clay soil. Generally speaking, the longer it takes for contaminant to move from beneath the landfill to the point of interest, the more diffusion that can occur and hence the lower the concentration at that point. The time interval between when the contaminant is released into the aquifer and when it arrives at a point outside the landfill is primarily dependent on the base velocity v_b. Thus the lower the value of v_b, the greater the amount of attenuation that will occur before the contaminant reaches point x. However, as previously noted, decreasing v_b will increase the concentration of contaminant beneath the landfill. Because of these conflicting mechanisms, there is a critical velocity at which the maximum peak concentration at any particular point will be reached. For the example case considered here, this critical velocity is about 1 m/a. Values of v_b either larger or smaller than 1 m/a will result in a smaller peak concentration in the aquifer. This has important practical consequences since it implies that to design for either the maximum or minimum expected base velocity in the aquifer is not necessarily conservative.

Effect of dispersion in the aquifer

The theory permits consideration of both horizontal advection and dispersion in the aquifer. In the analysis of the previous section, it was assumed that $D_H = 0$. Analyses were also performed for a range of values of velocity v_b and coefficient of dispersion D_H. In general, it was found that for velocities v_b of 1 m/a or greater, dispersion tended to reduce the maximum base concentration, although the effect was relatively small even for high values of D_H as shown for $v_b = 1$ m/a in Fig. 5.

For small advective velocities (i.e., less than 1 m/a), a high dispersion coefficient may have a significant effect on the peak concentration both beneath the landfill and at points outside the landfill. For example, Fig. 5 shows the variation in the peak concentration in the aquifer with D_H at two points ($x = 100$ m, $x = 400$ m) for a base velocity $v_b = 0.1$ m/a. For these low velocities, lateral dispersion in the aquifer reduces to the peak base concentration beneath the edge of the landfill and the effect
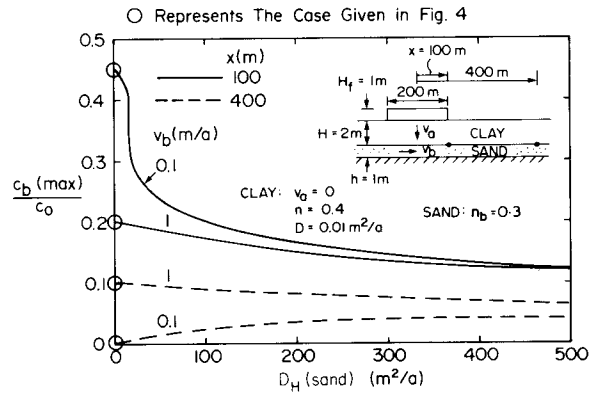
is quite significant for values of D_H between 0 and 50 m^2/a. At points remote from the landfill, lateral dispersion in the aquifer can lead to a modest increase in the predicted peak base concentration. This is because for low v_b increasing D_H increases the rate of mass transport through the aquifer and reduces the amount of diffusion that can occur into the adjacent clay layers. The magnitude of D_H encountered in the field is highly variable, although for $v_b = 0.1$ the expected range would be 0.01–20 m^2/a and most probably less than 3 m^2/a. These values could be up to an order of magnitude higher for $v_b = 1$ m/a.

As might be expected, the peak concentration at a point in the aquifer for $v_b = 0.1$ and 1 m/a tends towards a single value as D_H becomes very large and dominates over the effect of advection.

Because of nonhomogeneities present within most aquifers, the coefficient of dispersion in the sand (D_H) is a difficult parameter to determine and there will always be considerable uncertainty regarding the precise value to use in a design. When considering concentration beneath the landfill, it is conservative to perform the analysis for $D_H = 0$. When considering points outside the landfill, analysis may be performed for $D_H = 0$ and the maximum reasonable value of D_H. For low advection velocities, the analysis for the high D_H will be critical; for moderate and high advection velocities, the analysis for $D_H = 0$ will be conservative.

Concentration profiles in the clay

To this point, attention has been restricted to the concentrations within the aquifer, since this is of primary concern in the design of a landfill separated from an aquifer by a clay layer. However, the concentration profiles within the clay can also be determined as illustrated in Fig. 6. Beneath the landfill (Fig. 6a), the mass transport is predominantly downwards (although clearly some horizontal migration occurs near the edge of the landfill) and the concentrations decrease with depth. For the case examined, the surface concentration within the landfill has reduced to about 25% of the original value after 300 years. This reduction of concentration is entirely due to mass transport into the soil by diffusion (the effect would be greater if advection in the clay were also considered).

At locations remote from the landfill, the mass transport is also predominantly vertical but in this case it is upwards from the aquifer into the clay as shown in Fig. 6b.

Comparison of results from 1-D and 2-D analyses
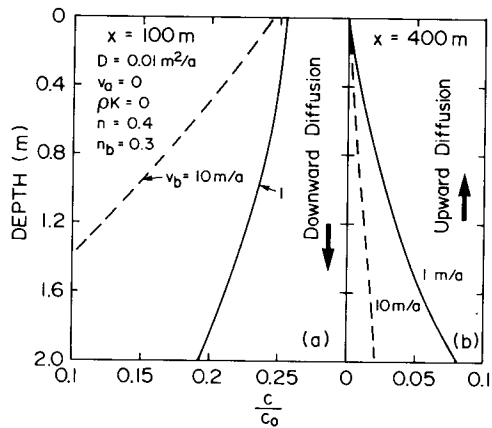
In problems analyzed by the authors, the computation

FIG. 6. Concentration profile in the clay barrier at two positions ($t = 300$ years): (a) $x = 100$ m; (b) $x = 400$ m.

involved in determining concentrations using the 2-D analysis described in this paper is small, although not insignificant, compared with that involved in alternative finite element analyses. Rowe and Booker (1985a) have previously developed a 1-D method of analysis that can be run on a microcomputer and that substantially reduces the amount of computation in comparison with the 2-D analysis. This then raises the question as to when it is desirable to use the full 2-D formulation and when it is adequate to use the simpler 1-D approach.

Clearly, the 1-D approach is ideally suited for truly one-dimensional problems such as the migration of contaminant in laboratory column/model tests (see Rowe et al. 1985b). The 1-D approach described by Rowe and Booker (1985a) assumes 1-D vertical transport within the clay while allowing for a reduction in concentration in the landfill as a result of mass transport into the clay. This 1-D approach also allows consideration of horizontal flow in an aquifer beneath the landfill (at a base velocity v_b) but in doing so assumes that the concentration is uniform both laterally and vertically within the aquifer directly beneath the landfill. This is not strictly correct and thus the 1-D analysis can be expected to underestimate the maximum concentration at the edge of the landfill when there is advective flow in the aquifer. The magnitude of the potential error can be assessed by comparing the results from the 1-D and 2-D analyses.

Figure 7 shows the variation in the concentration in the sand layer as a function of time calculated using the 1-D and 2-D analyses for a base flow velocity $v_b = 1$ m/a. In this comparison, the concentration from the 2-D analysis is evaluated at the downstream edge of the landfill $x = 100$ m and is the maximum concentration beneath the landfill at any time t. Comparing the results of the 1-D and 2-D analyses shows that the 1-D calculation gives a reasonable indication of both the magnitude of the peak concentration and the time at which the peak concentration occurs. The discrepancy is greater for no adsorption ($\rho K = 0$) where the 1-D analysis underestimates the peak concentration by 20% and overestimates the time required to reach the peak by 25%. This figure demonstrates that the simpler 1-D approach captures the form of the variation in concentration with time and for many practical problems an error of 25% would be acceptable in preliminary calculations.

Figure 8 shows the variation in the maximum peak concentration determined from 1-D and 2-D analyses for a range of base velocities. It is seen that the 1-D analysis consistently underestimates the maximum concentration (as would be expected) but
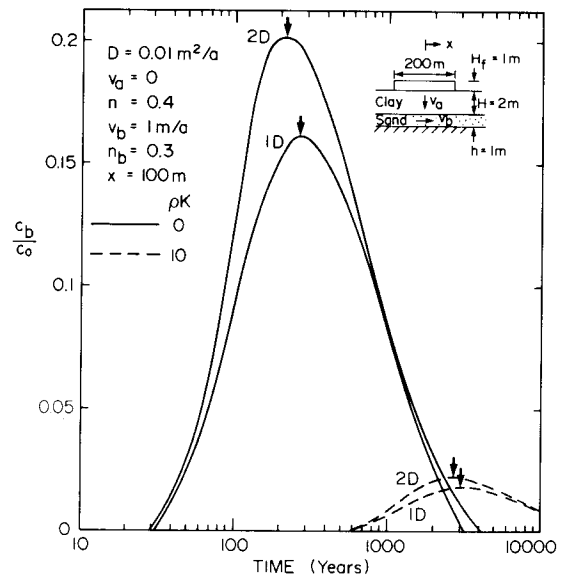

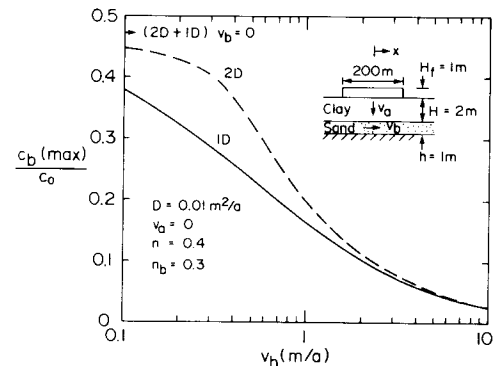FIG. 7. Variation in base concentration with time, 1-D vs. 2-D analysis.

FIG. 8. Variation in maximum base concentration with base velocity, 1-D vs. 2-D analysis.

that in general the error is quite small. The maximum error of 30% occurs for base velocities between 0.3 and 0.5 m/a. Similarly, it was found that the 1-D analysis also gives a reasonable estimate of the time required to attain the peak base concentration beneath the landfill.

The total mass transported into the soil from the landfill and the total mass transported from the clay into the aquifer directly beneath the landfill (i.e., -100 m $\leq x \leq 100$ m) as calculated from both the 1-D and 2-D analyses are given in Table 1. It can be seen that the 1-D analysis gives a very good indication of the mass transported into the soil for all cases and a quite reasonable indication of the mass transported into the aquifer directly beneath the landfill (i.e., -100 m $\leq x \leq 100$ m). The error in the total mass into the aquifer calculated using the 1-D approach is greatest for moderate velocities (i.e., $v_b \approx 1$ m/a) and large times but even then the error is modest (e.g., at most, 10% for the case examined in Table 1).

It can be shown analytically that the total mass present in the aquifer at any time t, for any velocity v_b, is equal to the mass into the aquifer calculated from the 1-D solution for $v_b = 0$, i.e.

$$m(t) = \int_{-\infty}^{\infty} \int_0^t f_z(x,H,\tau)d\tau dx = L \int_0^t f_1(H,\tau)d\tau$$

in which $f_1(H,\tau)$ is the flux into the aquifer at time τ determined

TABLE 1. Mass transport into the soil for $L = 200$ m, $H_f = 1$ m, $v_a = 0$,
$D = 0.01$ m^2/a, $\rho K = 0$

Time t(Year)	Base velocity v_b (m/a)	Dimensionless mass $(m/c_0 h^3)$				
		Mass into the soil from landfill up to time t		Mass into the aquifer directly beneath the landfill up to time t (i.e. $-L/2 \le x \le L/2$)		Mass retained in the aquifer at time t (i.e. $-\infty < x < \infty$)
		1D	2D	1D	2D	
100	0	66	66	7.2	7.2	7.2
	1	66	66	8.5	8.8	7.2
	10	66	66	11.7	12.3	7.2
1000	0	105	105	28.5	28.5	28.5
	1	150	157	122	135	28.5
	10	165	167	150	152	28.5

from the 1-D solution for $v_b = 0$; $f_z(x,H,\tau)$ is the flux into (or out of) the aquifer at time τ and position x from the 2-D solution for the actual value of v_b.

This analytic result is confirmed numerically in Table 1. Notice that the total mass present in the aquifer ($-\infty \le x \le \infty$) at any time t is less than the total mass that has passed into the aquifer directly beneath the landfill ($-L/2 < x < L/2$) up to this time t. The difference between these masses is due to diffusion from the aquifer back into the clay outside the landfill (i.e., $x < -L/2$, $x > L/2$). At large time the mass of the contaminant that has diffused into the clay is quite large. It is also interesting to note that the total mass remaining in the aquifer at any given time is independent of the base velocity v_b (although the total mass that has, at some time, moved into the base does depend on v_b).

Application

The analysis developed in the theory section can be readily programmed (Rowe et al. 1985a). The major computational effort involved is associated with the numerical inversion of the Laplace and Fourier transforms for the times and locations of interest. The Laplace transform can be inverted using a very efficient scheme proposed by Talbot (1979). Using this approach an accuracy of the order 10^{-6} and 10^{-10} can be achieved using 11 and 18 sample points respectively. For many practical problems an accuracy of 10^{-6} is adequate (since there is no time-marching procedure involved). The Fourier transform can be efficiently inverted using 20-point Gauss quadrature. The width and number of integration subintervals that are needed to achieve a reasonable accuracy (say 0.1%) depend somewhat on the geometry and properties of the problem under consideration. These parameters can be determined from a few trial calculations for a representative point and time of interest. Similarly, it should be noted that numerical experiments are also required to determine an appropriate finite element mesh and time integration procedure if alternative finite element (or finite difference) codes are used. The authors have obtained excellent agreement between calculations on this problem using a finite element program and a program that implements the theory described above.

The theory developed here for two-dimensional conditions assumes no mass transport in the third direction. The theory can, in fact, be readily extended to the three-dimensional case but the computational effort in this, as with any other three-dimensional

analysis, is quite large. Provided reasonable engineering judgement is exercised in selecting appropriate cross sections for analysis, it is considered that the two-dimensional analysis will be adequate for the vast majority of practical applications.

Clearly, if one is concerned with concentrations at points outside the landfill, then a two-dimensional analysis is necessary. Similarly, if an accurate distribution of concentration is required beneath the landfill, then a 2-D analysis is also appropriate. However, the results of this study also suggest that the simpler 1-D solution proposed by Rowe and Booker (1985a) may be useful for

(a) identifying when significant concentrations are to be expected in the aquifer directly beneath the landfill;

(b) providing an initial estimate of the magnitude of the peak concentration (and its time of occurrence) in the aquifer beneath the landfill, thereby minimizing the amount of computation required to get accurate values for these quantities from a subsequent 2-D analysis.

Conclusions

A technique for the analysis of 2-D pollutant migration through a soil layer of finite depth has been presented. This formulation permits consideration of the depletion of contaminant in the landfill with time as well as the effect of a flushing velocity in a permeable stratum beneath the clay layer.

A limited parametric study was performed to demonstrate some of the important features that arise from considering the finite mass of contaminant and a flushing velocity in an aquifer beneath the clay liner. For a finite mass of contaminant and the range of parameters considered, it may be concluded that:

1. The concentration of pollutant at any point will reach a maximum value at some time t_{max}, and will decrease for greater times. This maximum value can be used in design to ensure that the contamination of the groundwater never exceeds a specified level.

2. Diffusion of contaminant from the aquifer into the surrounding clay will provide natural attenuation of contaminant in the aquifer. The maximum concentration reached at any point outside the boundaries of the landfill will never reach the maximum value at the edge of the landfill. This phenomenon may be used in specifying the buffer zone required around a landfill to ensure the groundwater quality outside the buffer is never degraded below allowable levels owing to contamination from the landfill.

3. It was shown that there is a critical base velocity at which the maximum concentration of contaminant at a point in the aquifer away from the landfill will result. At velocities either larger or smaller than the critical velocity there will be a smaller peak concentration in the aquifer at the point of interest. Thus, to design for only the maximum and minimum expected velocities in the aquifer is not necessarily conservative, and a range of values between these limits should be considered.

4. Except for the case where the dispersion coefficient is very high or the advective velocity is very low, advection is the predominant mechanism for mass transport within the aquifer.

5. Comparison of results for 1-D and 2-D analyses indicates that the 1-D solutions may provide an adequate preliminary estimate of the contaminant concentration in a thin aquifer beneath a clay layer.

Acknowledgements

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List of symbols

$c = c(x,z,t)$	concentration of contaminant at any point (x,z) within the clay layer $(0 \leq z \leq H)$ at time t
$c_T = c(x,0,t)$	concentration of contaminant at the top of the clay layer $(z = 0)$
$c_b = c(x,H,t)$	concentration of contaminant within the base aquifer at position (x,H)
$c_b(\text{max})$	maximum concentration of contaminant that can be reached at position (x,H) in the base aquifer
c_{LF}	concentration of contaminant within the landfill
c_0	initial concentration of contaminant within the landfill
$\bar{c}$	Laplace transform of c
C	Fourier transform of c
D_{xx}, D_{zz}	coefficient of hydrodynamic dispersion (including diffusion and mechanical dispersion) in the x and z cartesian directions
D	isotropic coefficient of hydrodynamic dispersion
D_H	coefficient of hydrodynamic dispersion in the horizontal plane in the base aquifer
$f_x = f_x(x,z,t)$	the flux in the x-direction
$f_z = f_z(x,z,t)$	the flux in the z-direction
$f_T = f_z(x,0,t)$	the vertical flux entering the top of the clay layer at position $(x,0)$
$f_b = f_z(x,H,t)$	the vertical flux entering the base aquifer at position (x,H)
$\bar{f}_x, \bar{f}_z, \bar{f}_T, \bar{f}_b$	Laplace transforms of f_x, f_z, f_T, f_b
H_f	equivalent height of leachate (volume of leachate per unit plan area)
H	thickness of clay layer
h	thickness of aquifer
K	distribution coefficient
L	width of the landfill parallel to the velocity v_b
m	mass of contaminant
n	porosity of the clay layer
n_b	porosity of the base aquifer
t, τ	time
v_x, v_z	horizontal (x) and vertical (z) components of the seepage velocity
v_a	apparent (Darcy, superficial) vertical velocity $= nv_z$
v_b	apparent (Darcy, superficial) horizontal velocity in the base aquifer
x	horizontal distance from the centre of the landfill
z	vertical distance below the base of the landfill
ρ	bulk density of the soil solids